U. S. Department of Agriculture
Soil Conservation Service
Engineering Division

SOIL MECHANICS NOTE NO. 5

FLOW NET CONSTRUCTION AND USE

October, 1973

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I. Purpose and scope

The flow net is an excellent tool for evaluating the effects of seepage when adequate knowledge of embankment and/or foundation conditions is available. This note contains guides for the construction and use of flow nets. A number of illustrations are included to show how features such as cutoffs, drains, and impervious upstream blankets affect seepage.

II. Definitions

Definitions are mainly from ASTM Designation: D653-67. When definitions include units of measurement, the applicable units are indicated by capital letters in parentheses, as follows:

F = force, such as pound, gram

L = length, such as foot, centimeter

T = time, such as second, day

D = dimensionless

Positive exponents designate multiples in the numerator, whereas negative exponents designate multiples in the denominator.

A. Anisotropic mass - with respect to flow, a mass having different flow properties in different directions at a given point.

B. Aquifer - a water-bearing formation that provides a ground water reservoir.

C. Coefficient of permeability (permeability), k (LT^{-1}) - the rate of discharge of water under laminar flow conditions through a unit cross-sectional area of a porous medium under a unit hydraulic gradient and standard temperature conditions (usually 20° C).

This Note prepared by Robert E. Nelson and Donald M. Sundberg with comments by Rey S. Decker, Head, Soil Mechanics Unit, and Regional Soil Engineers.

D. Curvilinear square - any figure that has four sides and the same dimensions in the two primary directions. Also, any figure that can be sub-divided into three squares and a remaining figure similar in shape to the original is called a square.

E. Discharge velocity, v (LT^{-1}) - the rate of discharge of water through a porous medium per unit of total area perpendicular to the direction of flow.

F. Equipotential line - the line along which water will rise to the same elevation in piezometric tubes.

G. Flow channel - the portion of a flow net bounded by two adjacent flow lines. The number of flow channels in a flow net is denoted by N_f.

H. Flow line - the path that water follows in its course of seepage under laminar flow conditions.

I. Flow net - a graphical representation of flow lines and equipotential lines used in the study of seepage phenomena.

J. Homogeneous mass - a mass that exhibits essentially the same physical properties at every point throughout the mass.

K. Hydraulic gradient, i (D) - the loss of hydraulic head per unit distance of flow, dh/dL.

L. Hydrostatic pressure, u_o (FL^{-2}) - the pressure in a liquid under static conditions; the product of the unit weight of the liquid and the difference in elevation between the given point and the free water elevation.

Excess hydrostatic pressure, u (FL^{-2}) - the pressure that exists in pore water in excess of the hydrostatic pressure.

M. Isotropic mass - with respect to flow, a mass having the same property of flow in all directions.

N. Laminar flow - flow in which the head loss is proportional to the first power of the velocity.

O. Phreatic line (seepage line) - the upper free water surface of the zone of seepage.

P. Piezometer - an instrument for measuring pressure head.

Q. Piezometric surface - the surface at which water will stand in a series of piezometers.

R. Potential drop, Δh (L) - the difference in pressure head between two equipotential lines. The number of potential drops in a flow net is denoted by N_d.

S. Seepage (percolation) - the movement of gravitational water through the soil.

T. Seepage force, J (F) - the force transmitted to the soil grains by seepage.

U. Seepage velocity, v_s (LT^{-1}) - the rate of discharge of seepage water through a porous medium per unit area of void space perpendicular to the direction of flow.

V. Shape factor, $ (D) - a characteristic of a flow net which is independent of the permeability and the total head loss; the ratio N_f/N_d.

W. Transformed flow net - a flow net whose boundaries have been properly modified (transformed) so that a net consisting of curvilinear squares can be constructed to represent flow conditions in an anisotropic porous medium.

X. Uplift - the upward water pressure on a structure -- unit symbol, u (FL^{-2}) and total symbol, U (F).

III. General

The graphical construction of flow nets is useful in (a) developing a sense of how water movement takes place in porous media, (b) determining the effects of seepage on engineering works and natural slopes, and (c) evaluating the effects of alternate seepage control measures. Reasonable estimates of discharge quantities can be made from rather crude flow nets. However, more refined nets are needed for estimating seepage pressures and gradients, as these estimates depend on the manner in which pressures are distributed and dissipated.

Although flow through soil and rock materials is three dimensional, most cases can be represented in two dimensions (a drawing limitation) considering the third dimension to be a unit of width perpendicular to the direction of flow. Three-dimensional studies have been made with sand models or electrically conductive liquid models. For certain three-dimensional cases (not covered herein), two-dimensional flow nets can be combined with analytical procedures for analysis of seepage through abutments (as developed by A. Casagrande and R. Lo and given in R. Lo's Doctoral Thesis "Steady Seepage with Free Surface" - Harvard University - 1969).

The heterogeneous or stratified and lensed nature of soil deposits and bedrock materials generally makes it necessary to simplify the flow system for analysis. Often, layers with similar permeability rates can be combined so that the total number of layers to consider is minimized. Anisotropic or stratified layers can be transformed, as discussed in Section IV-D, to simplify the problem. It is emphasized that simplifying assumptions in seepage analysis impose requirements for judgment in selection of design values as in other soil engineering analyses.

IV. Principles of flow nets

A. Assumptions

1. Materials are homogeneous and isotropic. (When materials are anisotropic, use transformation techniques that are discussed in Section IV-D.)

2. Materials are saturated (all voids are filled with water).

3. Materials and water are incompressible (no volume change - steady state flow).

4. Darcy's law, $v = ki$, applies.

B. Mathematical representation

The assumption that flow across an element of material does not change with time (steady state) can be represented in differential form as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{(Eq. 1)}$$

which is the equation of continuity with u, v, and w being components of discharge velocity. Expressing these velocity components in the Darcy form, $v = k\frac{h}{L}$, and substituting into Equation 1 results in the general differential equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0; \qquad \text{(Eq. 2)}$$

or in two dimensions:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad \text{(Eq. 3)}$$

Equation 3 represents two sets of curves that describe flow through materials -- one set being equipotential lines and the other set being flow lines.

C. The flow net

Properties of the flow net and equations for estimating discharge quantity, seepage pressure, and discharge gradient are presented in Figure 1. Figures 2 and 3 illustrate flow directions at entrance boundaries, exit boundaries, and across boundaries between soils of different permeabilities. These figures are particularly helpful in sketching flow nets because of the variations given.

D. Transformations

1. Anisotropic materials

Sections consisting of anisotropic materials must be transformed to equivalent isotropic sections. This is done by (1) decreasing dimensions in the direction of maximum permeability, k_{max}, leaving dimensions in the direction of minimum permeability, k_{min}, unchanged or (2) increasing dimensions in the direction of minimum permeability, k_{min}, leaving dimensions in the direction of maximum permeability, k_{max}, unchanged. Ratios are as follows:

a. To decrease dimensions in the direction of k_{max}, multiply them by $\sqrt{k_{min}/k_{max}}$. This is illustrated in Figure 4(b).

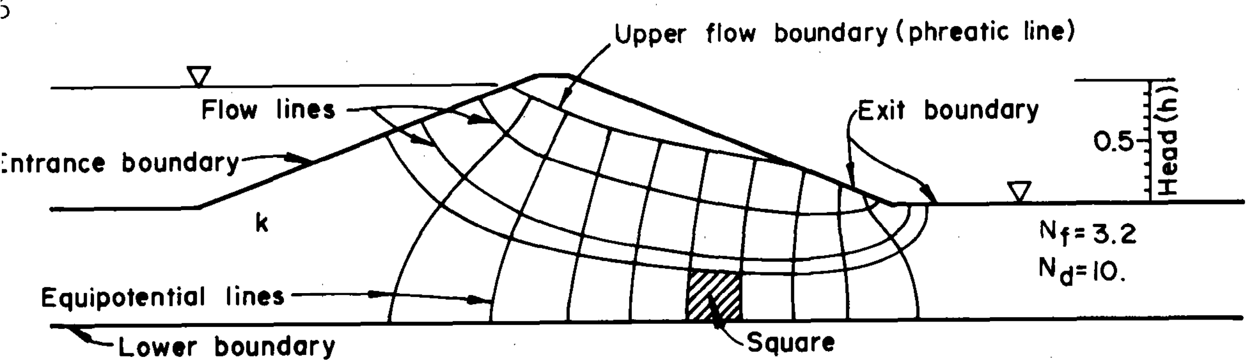
b. To increase dimensions in the direction of k_{min}, multiply them by $\sqrt{k_{max}/k_{min}}$.

The choice of transformation direction should be made so that the size of the transformed section is large enough to provide detail, yet small enough so as not to be unwieldy. For example, the section in Figure 4(b) was drawn to twice the size shown.

Permeability of the transformed section, which is isotropic, is

$$k' = \sqrt{(k_{min})\ (k_{max})} \qquad \text{(Eq. 5)}$$

The rate of discharge may be obtained directly from the flow net for the transformed section using k' instead of k in Equation 4a. The flow net must be transposed back to true scale for assessment of pressures and gradients. Figure 4(c) illustrates this process; horizontal dimensions in Figure 4(b) were multiplied by $\sqrt{k_{max}/k_{min}}$ to obtain the actual net.



(a) Nomenclature

Flow and equipotential lines intersect at right angles and are chosen to form curvilinear squares.

Flow quantity between all pairs of adjacent flow lines (in all flow channels) is the same.

Energy loss between all pairs of equipotential lines is the same.

Velocity and hydraulic gradient are a function of the spacing between flow and equipotential lines.

Lines within the net are smooth curves.

Squares at a discharge face exposed to the atmosphere may be incomplete.

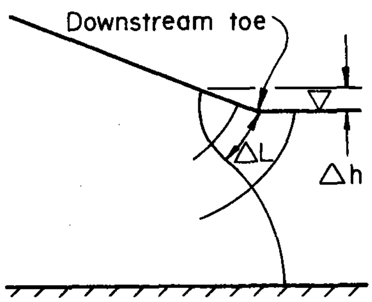
Equipotential lines intersect the phreatic line at equal increments of elevation.

(b) Properties

te of discharge (q) $= kh \dfrac{N_f}{N_d} = kh\,\$$ (Eq. 4a)

scharge gradient $(i_d) = \Delta h / \Delta L$ (Eq. 4b)

epage pressure $(p_s) = \dfrac{J}{A} = \dfrac{\gamma_w \, i_d \, V}{A} = \gamma_w \, i_d$ (Eq. 4c)



where $\Delta h = h/N_d$

ΔL = flow path length across square at discharge face (see adjacent sketch)

V = unit volume

A = unit cross sectional area

γ_w = unit weight of water

$\$$ = shape factor

(c) Equations

Figure 1. Nomenclature, properties, and equations relating to flow nets.

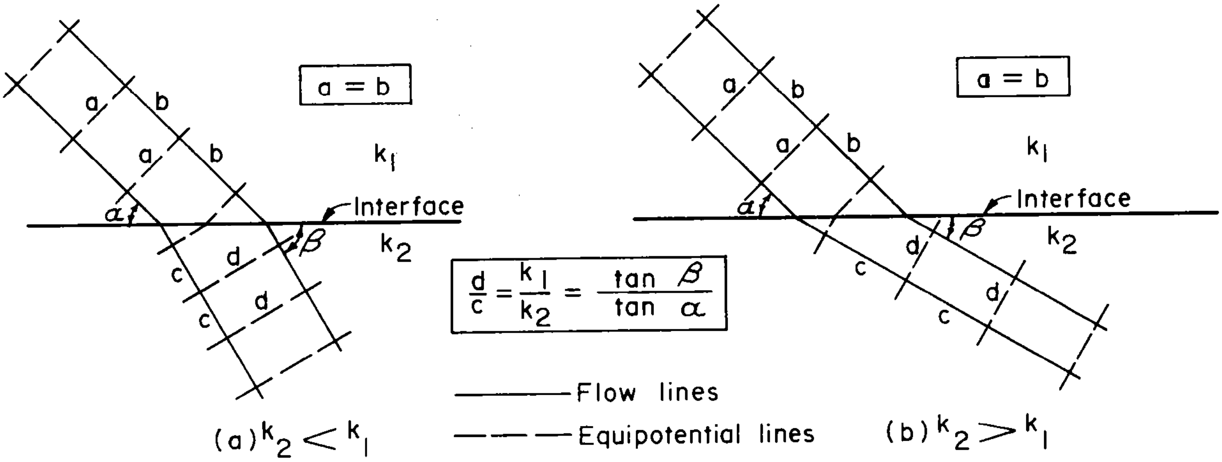


Figure 2. Deflections of flow lines at interfaces of soils having different permeabilities

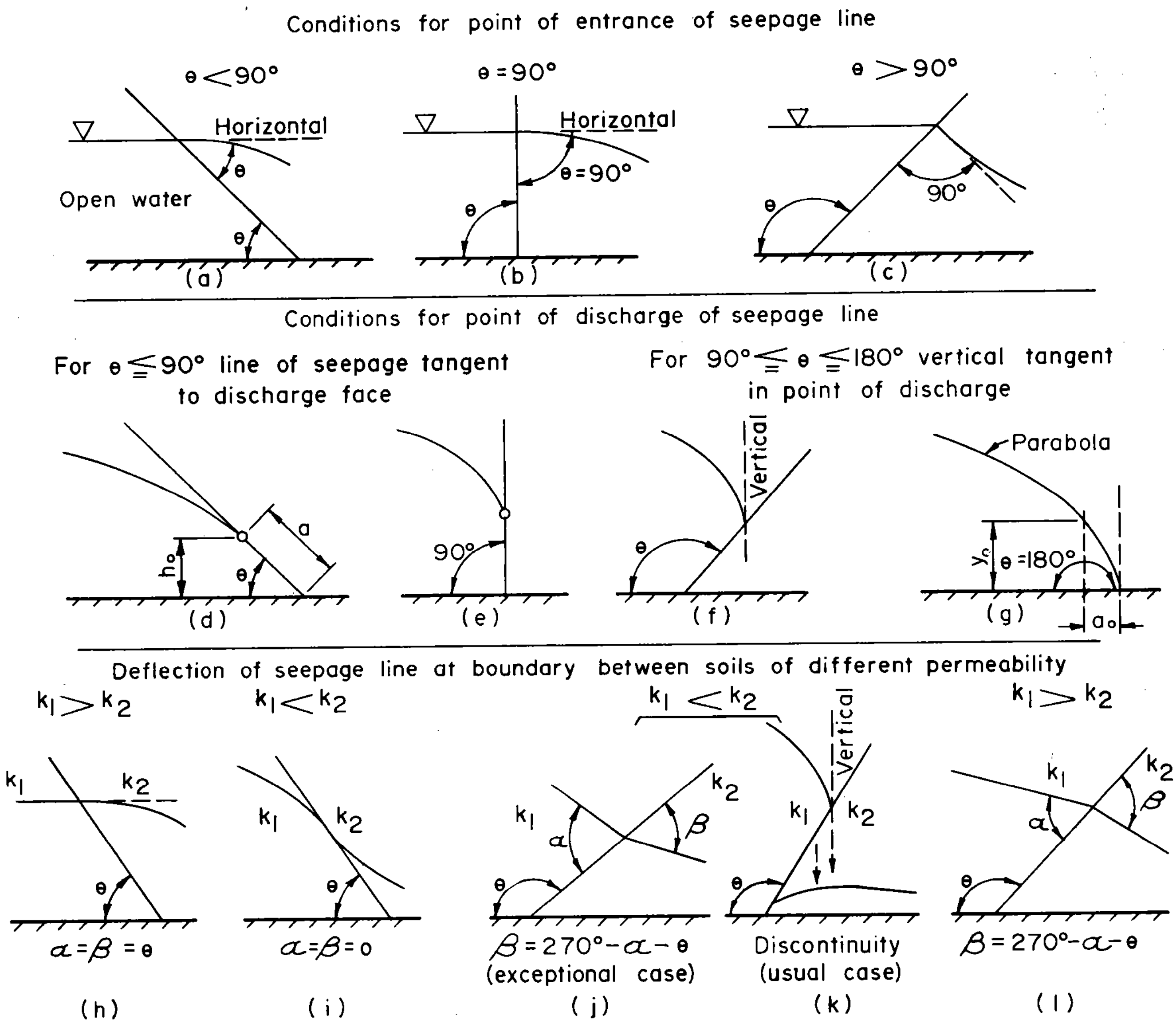


Figure 3. Entrance, discharge and transfer conditions of seepage lines

Adapted from The Journal Of The New England Water Works Association, Vol. 51, No. 2.

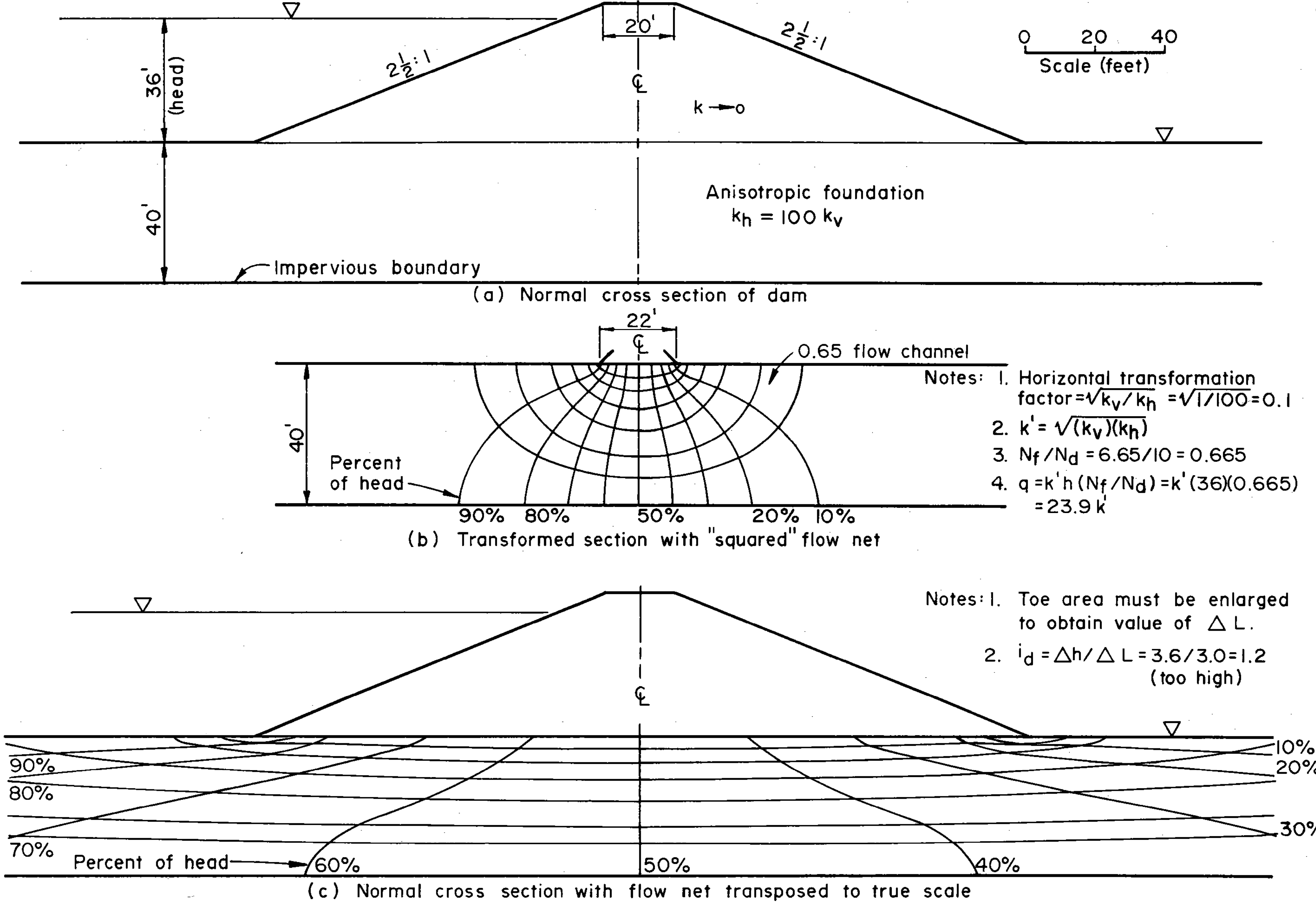


Figure 4. Example of technique for developing flow net for anisotropic foundation

2. Stratified materials

Most natural materials and many compacted soils are stratified, having layers of differing permeability. Transformation of cross sections representing stratified materials is a two-step process requiring that the permeability and the thickness of each layer be considered. Simplifying assumptions are that permeability and thickness of each layer being considered are constants.

a. Change the stratified section to an equivalent anisotropic section. The following equations adjust permeabilities in directions parallel and perpendicular to the bedding plane (see Figure 5(a) and (b)).

$$k_{max} = \frac{1}{d} (k_1 d_1 + k_2 d_2 + \ldots . + k_n d_n) \qquad \text{(Eq. 6)}$$

$$k_{min} = \frac{d}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \ldots . + \frac{d_n}{k_n}} \qquad \text{(Eq. 7)}$$

where d = total thickness of the stratified deposit

$d_1, d_2, \ldots d_n$ = thickness of the individual layers

$k_1, k_2, \ldots k_n$ = coefficient of permeability of the individual layers

k_{max} = overall coefficient of permeability parallel to the bedding plane

k_{min} = overall coefficient of permeability perpendicular to the bedding plane

b. Transform the anisotropic section to an isotropic section (see Section IV-D-1 and Figure 5(c)).

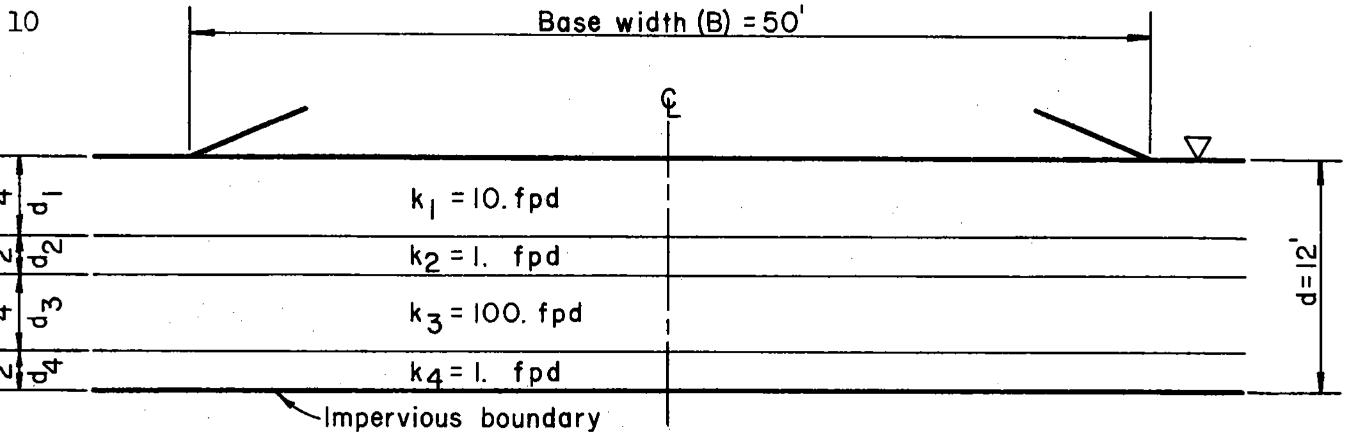
c. Construct the flow net on the isotropic section.

The rate of discharge may be obtained from this flow net, but it cannot be used to evaluate discharge gradient for the stratified materials.

V. Site information required

The following information is needed to evaluate site conditions and set up the "model" for flow net analysis:

A. Topographic maps of the site area.

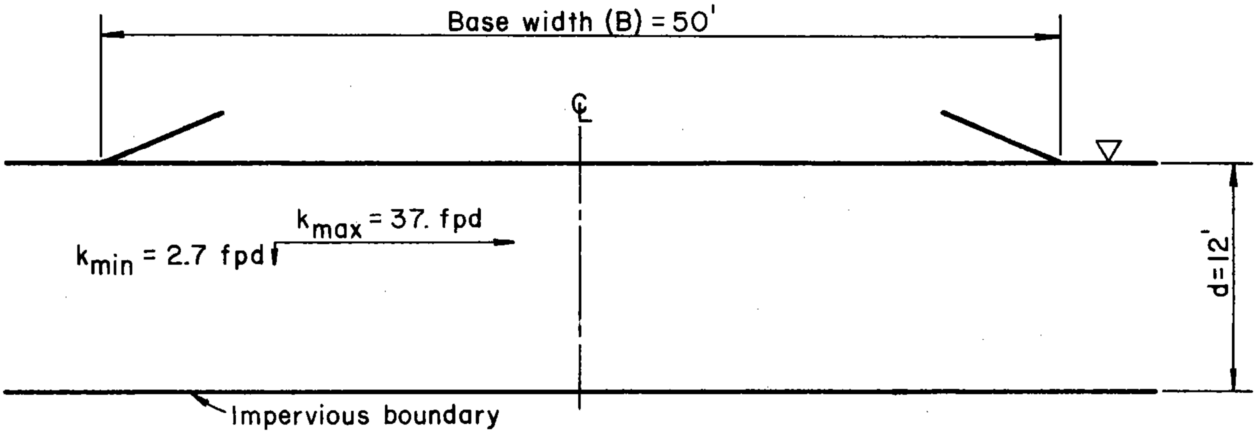


(a) Stratified section

Permeability values for the 12-ft. thick equivalent anisotropic section

From Eq. 6: $k_{max} = \frac{1}{12}\left[(10 \times 4)+(1 \times 2)+(100 \times 4)+(1 \times 2)\right] = \frac{1}{12} \times 444 = 37.\text{fpd}$

From Eq. 7: $k_{min} = \dfrac{12}{\frac{4}{10}+\frac{2}{1}+\frac{4}{100}+\frac{2}{1}} = \dfrac{12}{4.44} = 2.7 \text{ fpd}$

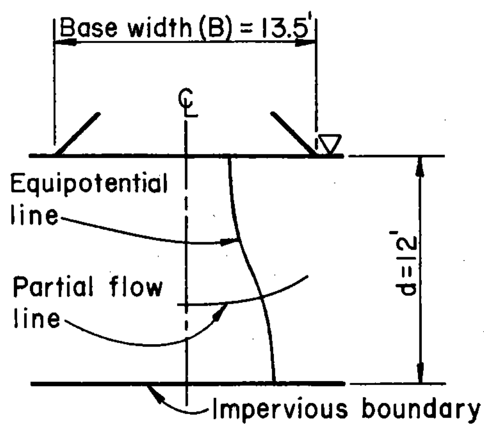


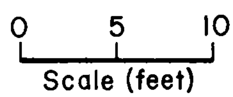
(b) Equivalent anisotropic section

Horizontal transformation factor for (c) =

$\sqrt{k_{min}/k_{max}} = \sqrt{2.7/37} = 0.27$

B=50 x 0.27 = 13.5'





(c) Isotropic section (transformed)

Figure 5. Transformation of a stratified section

B. Detailed geologic maps and sections of the site including reservoirs, channels, and any features that may be affected such as areas downstream and adjacent valleys.

C. Detailed logs and descriptions of all materials. Items such as gradation, soil structure, stratification, continuity of strata, artesian pressure, and moisture content are important.

D. Location, depth, gradient, and areal extent of water tables as may affect the proposed project.

E. Direction of ground water flow.

F. Pressure gradient in and from confined layers.

G. Permeability of all materials.

H. Surface water levels with the structure in operation.

I. Geometry of the structure.

VI. Flow net construction

Most of this section is devoted to flow net sketching. This method takes less time than methods utilizing models and is sufficiently accurate. Sketching materials are available in all Service design offices. Special equipment required for model construction is not.

A. Sketching

The suggestions that follow are particularly helpful to the beginner. They are quoted in part from "Seepage Through Dams" by A. Casagrande, Journal of the New England Water Works Association, June 1937, reprinted in "Contributions to Soil Mechanics 1925-1940", Boston Society of Civil Engineers, 1940.

1. "Use every opportunity to study the appearance of well constructed flow nets. When the picture is sufficiently absorbed in your mind, try to draw the same flow net without looking at the available solution; repeat this until you are able to sketch this flow net in a satisfactory manner."

2. "Four or five flow channels are usually sufficient for the first attempts; the use of too many flow channels may distract attention from the essential features."

3. "Always watch the appearance of the entire flow net. Do not try to adjust details before the entire flow net is approximately correct."

4. "Frequently there are portions of a flow net in which the flow lines should be approximately straight and parallel lines. The flow channels are then about of equal width, and the squares are therefore uniform in size. By starting to plot the flow net in such an area, assuming it to consist of straight lines, one can facilitate the solution."

5. "The flow net in confined areas, limited by parallel boundaries, is frequently symmetrical, consisting of curves of elliptical shape."

6. "The beginner usually makes the mistake of drawing too sharp transitions between straight and curved sections of flow lines or equipotential lines. Keep in mind that all transitions are smooth and elliptical or parabolic in shape. The size of the squares in each channel will change gradually."

7. "In general, the first assumption of flow channels will not result in a flow net consisting throughout of squares. The drop in head between neighboring equipotential lines corresponding to the arbitrary number of flow channels, will usually not be an integer of the total drop in head. Thus, where the flow net is ended, a row of rectangles will remain. For usual purposes, this has no disadvantages, and the last row is taken into consideration in computations by estimating the ratio of the sides of the rectangles."

   The partial drop should be located in an area where squares are nearly uniform in size. This makes it easier to estimate the fractional value of the partial drop. The ratio of the length to the width of these rectangles should be the same throughout the partial drop.

   When all flow paths are relatively short, it may be easier to begin sketching with an integer number of equipotential drops. The resulting flow net will usually contain a partial flow channel which should be located near the center of all flow channels, i.e. away from flow boundaries.

8. "A discharge face in contact with air is neither a flow line nor an equipotential line. Therefore, the squares along such a boundary are incomplete. However, such a boundary must fulfill the same condition as the line of seepage regarding equal drops in head between the points where the equipotential lines intersect."

In addition:

1. Use a good grade of unlined paper.

2. Draw boundaries in ink or overlay and sketch on a good grade of tracing paper.

3. Select a scale small enough to permit viewing the entire net at a glance.

4. Use a soft pencil and a good, soft eraser.

5. Rotate the drawing during sketching for perspective.

Figure 6 contains guides for line directions and intersection locations that help proportion the flow net. They apply to lines at extremities of the net and near boundary changes. These guides were presented in the course "Seepage and Ground Water Flow" by A. Casagrande, Harvard Graduate School of Engineering.

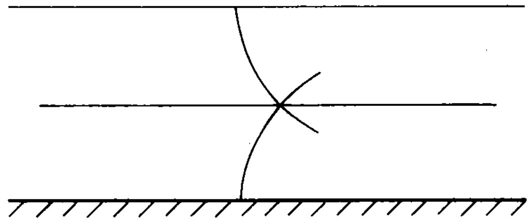
Although the flow net developed in Figure 7 is unique for a thin cutoff wall taken to the midpoint of a pervious stratum, the steps illustrated are helpful for other cases where flow is limited to foundation material. Often flow will be symmetrical about a vertical plane, and sketching can be limited to half of the net. Begin by selecting the number of flow channels (three in this case) and tick mark them on the vertical equipotential line beneath the cutoff as in Figure 7(a). Establish line directions near the cutoff bottom with the 2:1 guide from Figure 6(f) and then sketch the equipotential lines as in Figure 7(c). Adjust lines as necessary to "square" the net with 90-degree intersections.

Figure 8 illustrates how to develop a flow net beginning with an integer number of equipotential drops. Divide the net head into equal vertical increments (Δh) as in Figure 8(a). Sketch in a phreatic line and the equipotential lines as in Figure 8(b). The equipotential lines should intersect the phreatic line at 90 degrees at points where the head increments intersect the phreatic line. Sketch in the flow lines to intersect the equipotential lines at 90 degrees, "squaring" the net as in Figure 8(c). Usually several trial locations for the phreatic line are required to obtain a "square" net. The intersection of the upstream flow line and the upstream equipotential line should lie approximately on the bisector of the upstream slope angle in accordance with the guide in Figure 6(c). Flow enters the embankment perpendicular to the upstream slope and exits vertically into the drainage section. Note that, when the equipotential drops are arbitrarily selected as a whole number (six in this case), a fractional number of flow channels appears in the final net.

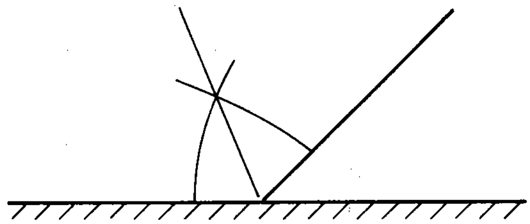
B. Electric analog models

Darcy's law for flow of water through a porous medium is similar to Ohm's law for flow of electric current through a uniform conductor.

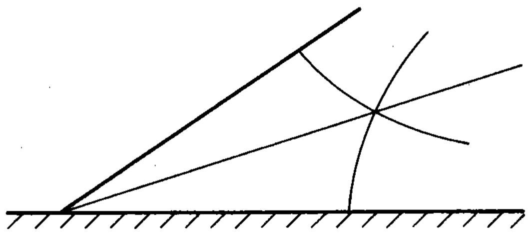
(a). For foundations, furthermost upstream and downstream flow lines and equipotential lines should intersect at or near the center of the pervious foundation.



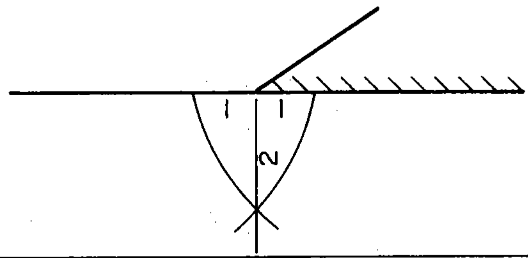
(b). The flow line and equipotential line nearest an angle should intersect on the bisector of the angle.



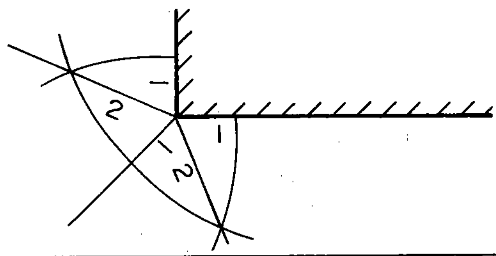
(c). Same as (b) except for an upstream toe on an impervious foundation.



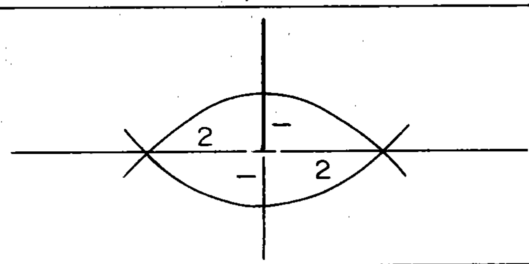
(d). 2:1 length ratios to establish shape of the "square" in a pervious foundation at the toe of an impervious fill.



(e). 2:1 length ratios used with angle bisectors to shape flow around an imbedded 90-degree angle.



(f). 2:1 length ratios to establish flow directions beneath a thin cutoff wall taken to the midpoint of the pervious stratum.



(g). Subdivide to check odd-shaped "squares". Resulting smaller odd-shaped "squares" should have the general shape of the one subdivided.

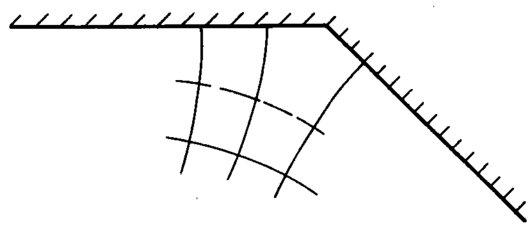
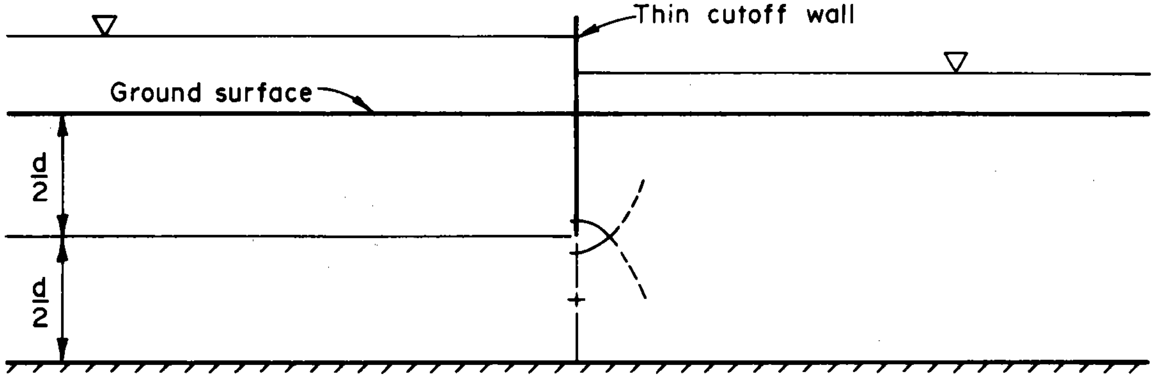
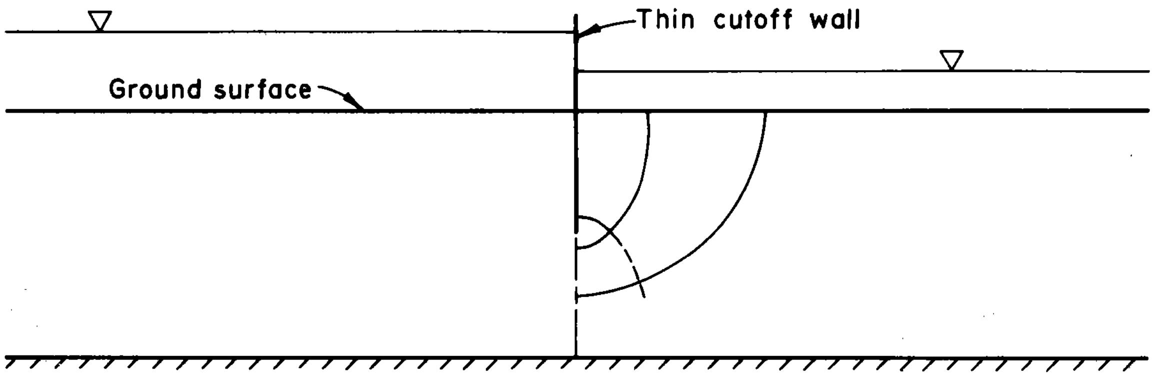


Figure 6. Guides for flow net construction.

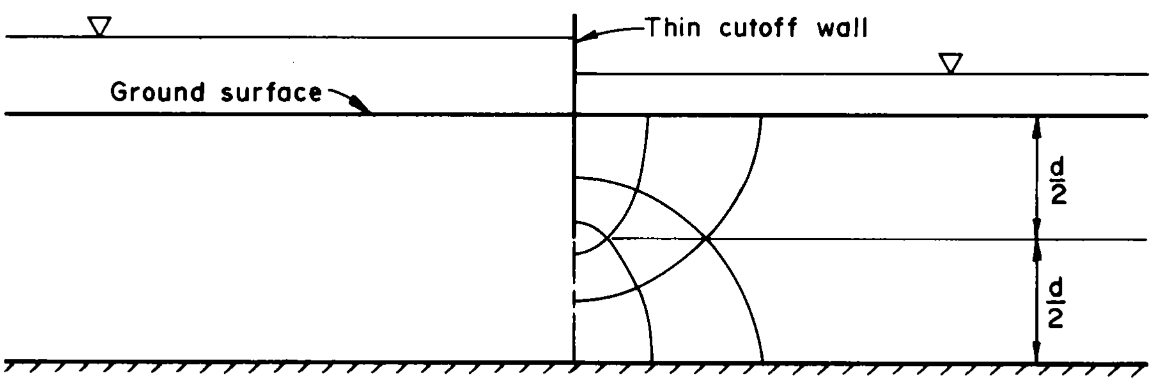
From: course titled "Seepage and Ground Water Flow" presented by A. Casagrande at Harvard University



(a)

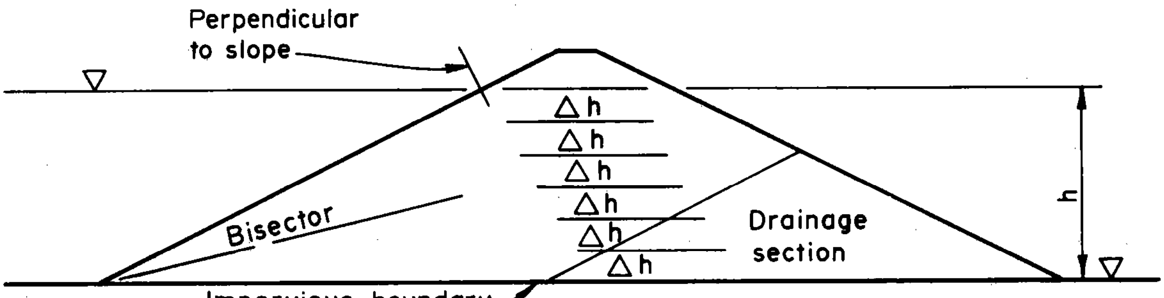


(b)

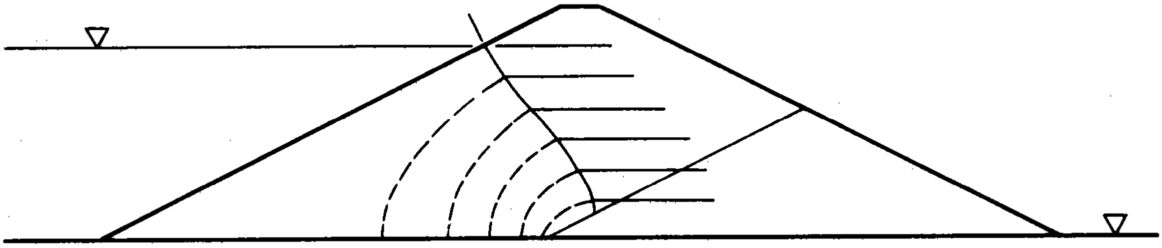


(c)

Figure 7. Sketching—flow limited to foundation.



(a)


(b)

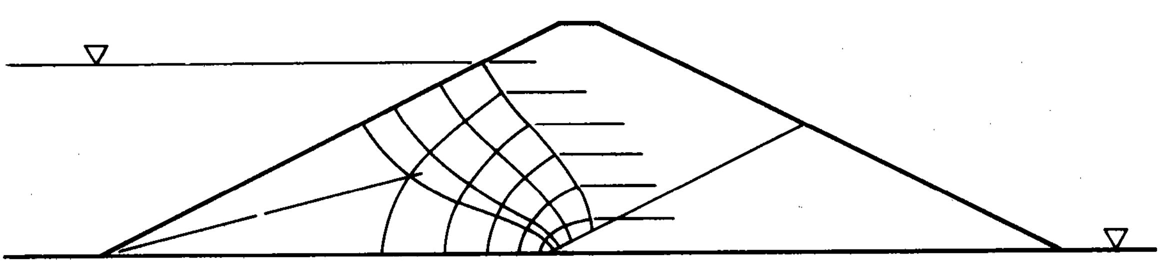

(c)

Figure 8. Sketching—flow limited to embankment

Darcy's law	Ohm's law
$q = \dfrac{k\ h\ A}{L}$	$I = \dfrac{k'\ V\ A'}{L'}$
where q = rate of water flow	where I = rate of current flow
k = permeability coefficient	k' = conductivity coefficient (reciprocal of resistance)
h = head drop	V = voltage drop
A = area of flow	A' = area of flow
L = length of flow path	L' = length of flow path

Electrical conductive models can be used to study steady-state flow problems because of this similarity. Solid and liquid conductive media are used for models. Thus far, work done in the Service has been limited to solid medium using (1) a conductive paper or (2) graphite-bentonite-water mixtures spray-painted on rigid insulating material. Resistor grid networks have been used successfully by other organizations.

Conductive paper is somewhat limited in usage because it is isotropic, or nearly so. Relatively simple anisotropic systems may be handled by transformation as discussed in Section IV-D.

A spray-painted model is used when the flow system has layers differing in permeability. Conductivity of the medium depends on the number of coats applied and the quantity of graphite in the mixture.

Figure 9 shows how electrical potentials are applied. It is convenient to express voltages as a percentage of the total voltage drop across the model. Direct current is generally used with solid conductive media.

Equipotential lines are usually developed from the model, as in Figure 9, and flow lines are sketched. It is possible to develop flow lines with the same equipment by changing the circuitry.

C. Sand models

A striking illustration of flow line development in a dam can be produced by the use of sand models. Such models can be built to represent any prototype, although they are usually limited to homogeneous embankments or zoned embankments consisting of two materials -- one in the core and the other in the upstream and the downstream shells.

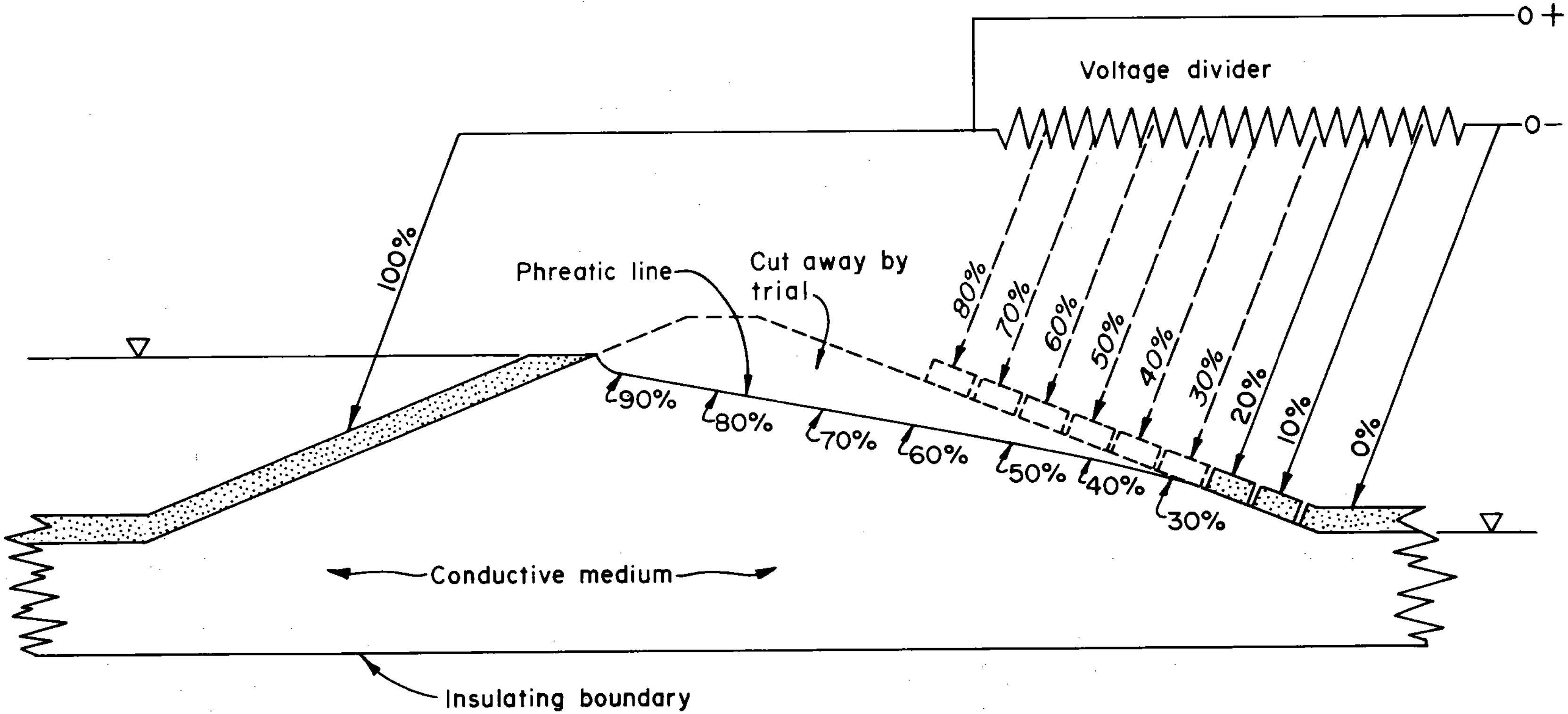


Figure 9. Model potentials in an embankment. (Steady seepage condition)

Figures 10 and 11 illustrate an homogeneous and a zoned model dam respectively as constructed at the Soil Mechanics Unit. The steady seepage condition is depicted in each case. After the embankment has saturated, dye crystals (potassium permanganate) were placed at random intervals on the upstream slopes. The resulting lines indicate the flow paths of water as it moves through the embankment.

Flow paths indicated by the dye can be copied onto a sketch. By adjustment of flow paths and addition of equipotential lines, the model data can be used to develop flow nets.

Like electric models, sand models only give the flow pattern for one condition, except that in sand models drains at the embankment-foundation interface may be installed at several locations and manipulated as desired. These drains may be closed, as in Figure 10(a), so that the flow lines egress on the downstream slope; or they may be opened. In Figure 10(b), the drain, located at c = 0.3b, is open and shows how flow lines in an isotropic embankment converge on the drain. (b = horizontal projection of downstream slope; c = horizontal distance from downstream crest of embankment to upstream edge of drain.)

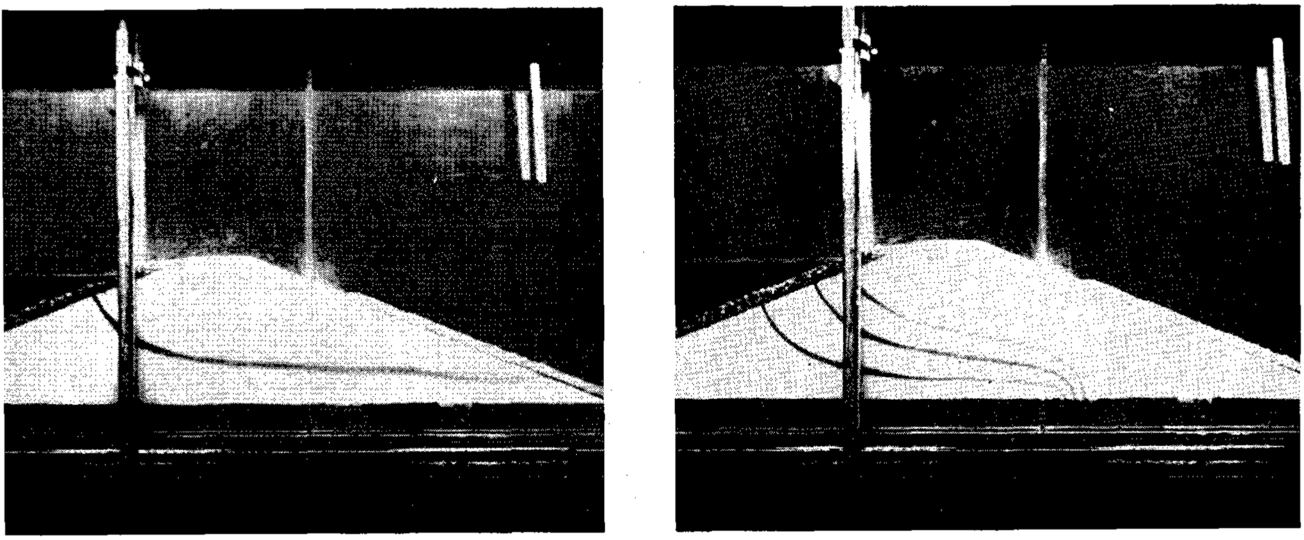
Six flow paths are illustrated in the zoned embankment in Figure 11. In Figure 11(a), the tailwater is at the ground surface. Flow lines are fully developed in the upstream shell and core; the downstream shell is dry except at the extreme bottom. (Note: The dark line higher up in the downstream shell is a residual water mark from a previous demonstration.) The tailwater has been raised in Figure 11(b) to about one-third the dam height to illustrate the change in flow lines with a change in head potential.

Sand models take considerable time to build. It is difficult to obtain isotropy in the cross section and, when zoned, a realistic variation of the permeability ratio of the core and the shell materials. Furthermore, it is difficult to obtain the top flow line when the reservoir is full due to capillary rise of the dyed water and resulting blob in the upper part of the embankment.

This type of model is especially useful for training purposes, as it produces a graphic picture of seepage.

D. Instrumentation

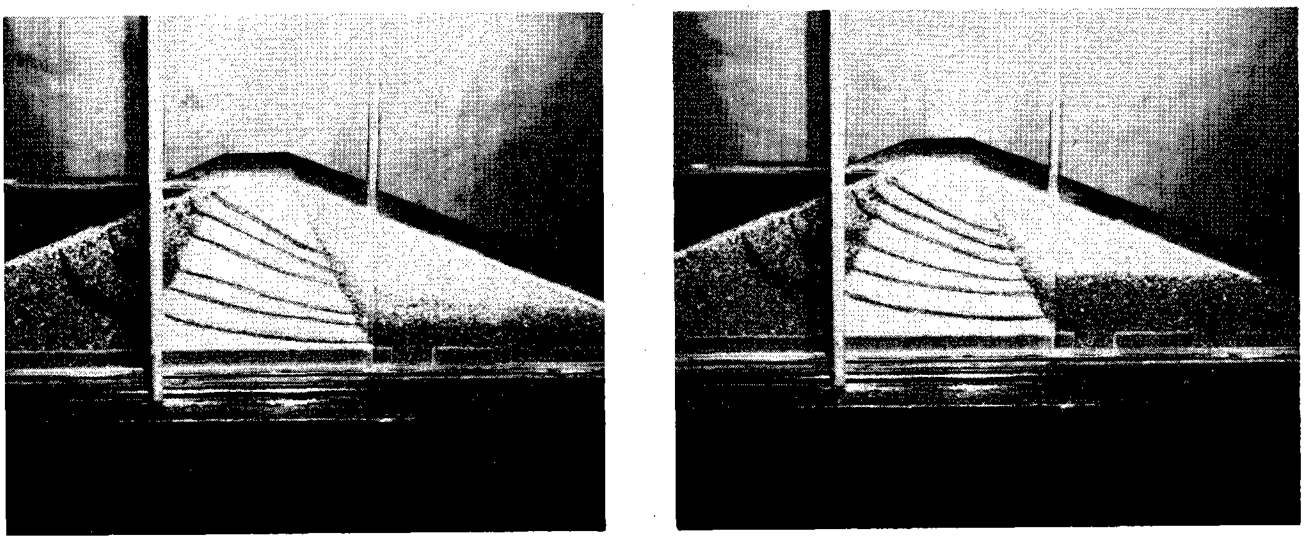
The techniques presented in the three preceding sections (A, B, and C) either represent theoretical flow nets or nets developed for specific situations prior to design.



(a) Drains closed

(b) Upstream drain (at c = 0.3b) open

Figure 10. Homogeneous sand model with headwater near top of embankment—impervious foundation.



(a) No tail water

(b) Tail water $\frac{1}{4}$ to $\frac{1}{3}$ height up backslope

Figure 11. Zoned sand model with headwater near top of embankment—impervious foundation.

Piezometers can be installed in earth and rock masses to determine actual flow conditions. Flow nets drawn from piezometric data are helpful in analyzing problems concerning subsurface flow. For this purpose, pressures must be monitored at several levels at a sufficient number of observation points to effectively define subsurface flow conditions.

VII. Use of the flow net

From an accurately constructed flow net, the rate at which water is seeping through a porous medium (within the boundaries of the flow net) can be determined; the seepage gradient can be computed; and the magnitude and direction of seepage forces or seepage pressure at any point may be determined.

A. Rate of seepage discharge

After construction of the flow net is complete, the number of flow channels (N_f) and the number of equipotential drops (N_d) are counted. Partial flow channels or partial drops are considered (e.g. N_f may equal 3.2). The shape factor ($) is calculated from the ratio of the number of flow channels to the number of equipotential drops (N_f/N_d). Then, rate of discharge is computed according to Equation 4a, Figure 1.

B. Seepage gradients

If one square (see Figure 1a) within the flow net is considered, the loss of head due to flow through the square is equal to the change in head from one equipotential line to the next (Δh). The gradient at any point is the loss in head between equipotential lines (Δh) divided by the distance between equipotential lines (ΔL) measured along the flow path. The path is considered to be midway between flow lines. The seepage gradient changes throughout the flow net because squares vary in size. The largest gradient occurs along the shortest flow path. The point of concern is usually where the gradient is the largest along the discharge face. Generally, this occurs at the downstream toe of a dam or at the point where the shortest flow path discharges. The discharge gradient (i_d) is calculated from Equation 4b, Figure 1.

C. Seepage forces and pressures

The seepage force (J) acting on any square in the flow net is the product of the gradient across the square, the unit weight of water, and the volume of the square. When the volume is unity, this force is numerically equal to the difference in head between the upstream side and the downstream side (Δh) times the unit weight of water. The seepage force is exerted on the porous medium by the water moving through it and acts in the direction of flow parallel to the flow lines at the point in question.

The seepage pressure (p_s) acting on any square is equal to the seepage force divided by the area of the square normal to the direction of flow. Seepage pressure is calculated from Equation 4c, Figure 1.

VIII. Selected references

A. "Seepage Through Dams" by Arthur Casagrande, Journal of the New England Water Works Association, June 1937, reprinted in Contributions to Soil Mechanics 1925-1940, Boston Society of Civil Engineers, 1940.

B. Seepage, Drainage and Flow Nets by Harry Cedergren, Wiley, 1967.

C. Theoretical Soil Mechanics by Karl Terzaghi, Wiley, 1943.

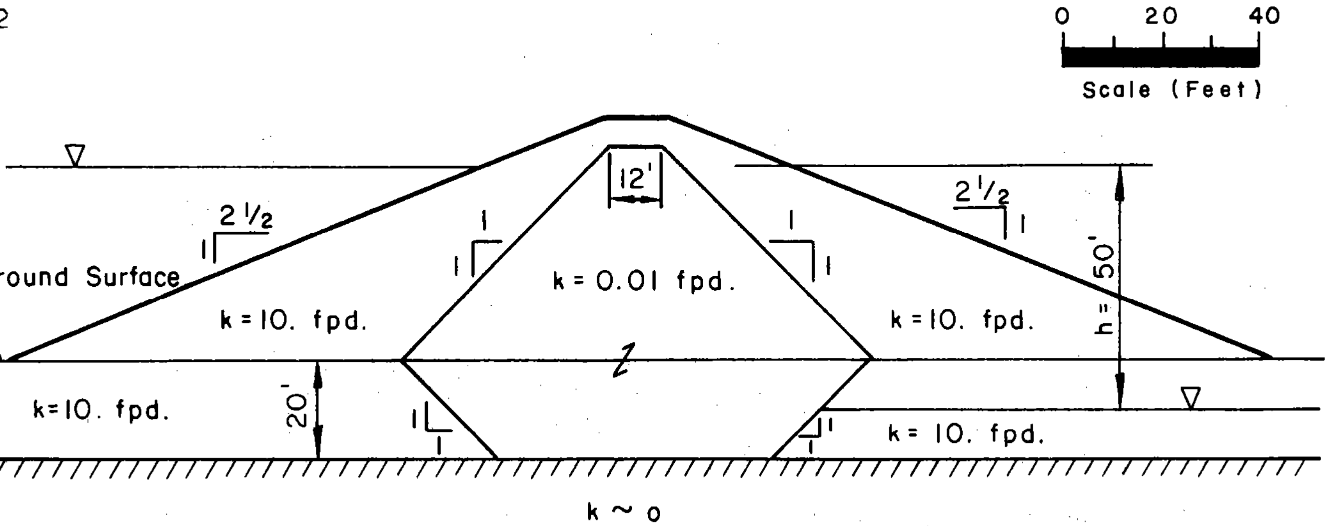
IX. Example problems

A. Example 1. Seepage loss and discharge gradient.

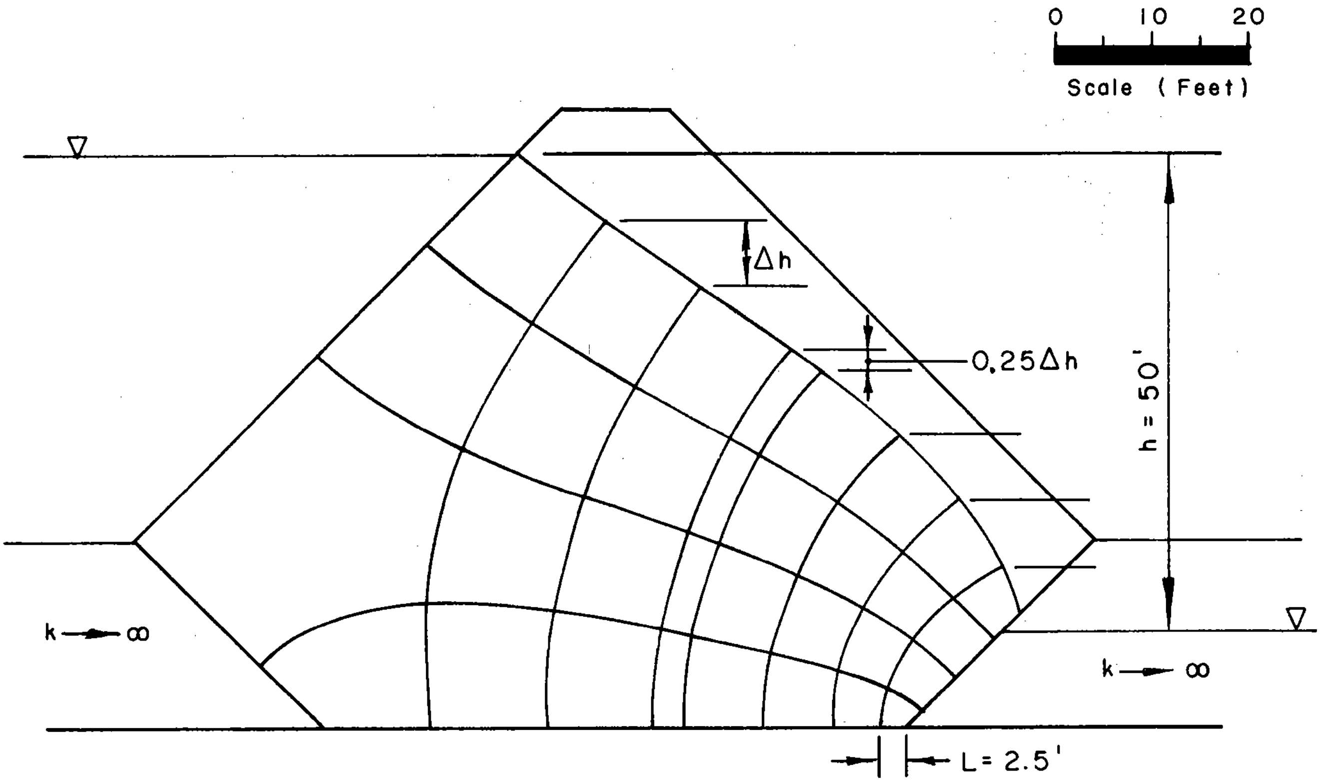
B. Example 2. Discharge to a drain.

C. Example 3. Effect of anisotropy on the phreatic line.

D. Example 4. Concrete drop spillway--base uplift and discharge gradient.



For the above storage dam, estimate the water loss per foot length of dam and the discharge gradient at the base of the cutoff. Since k of embankment shells and foundation is much more than that of the core and cutoff, construct the flow net only for the core and cutoff.

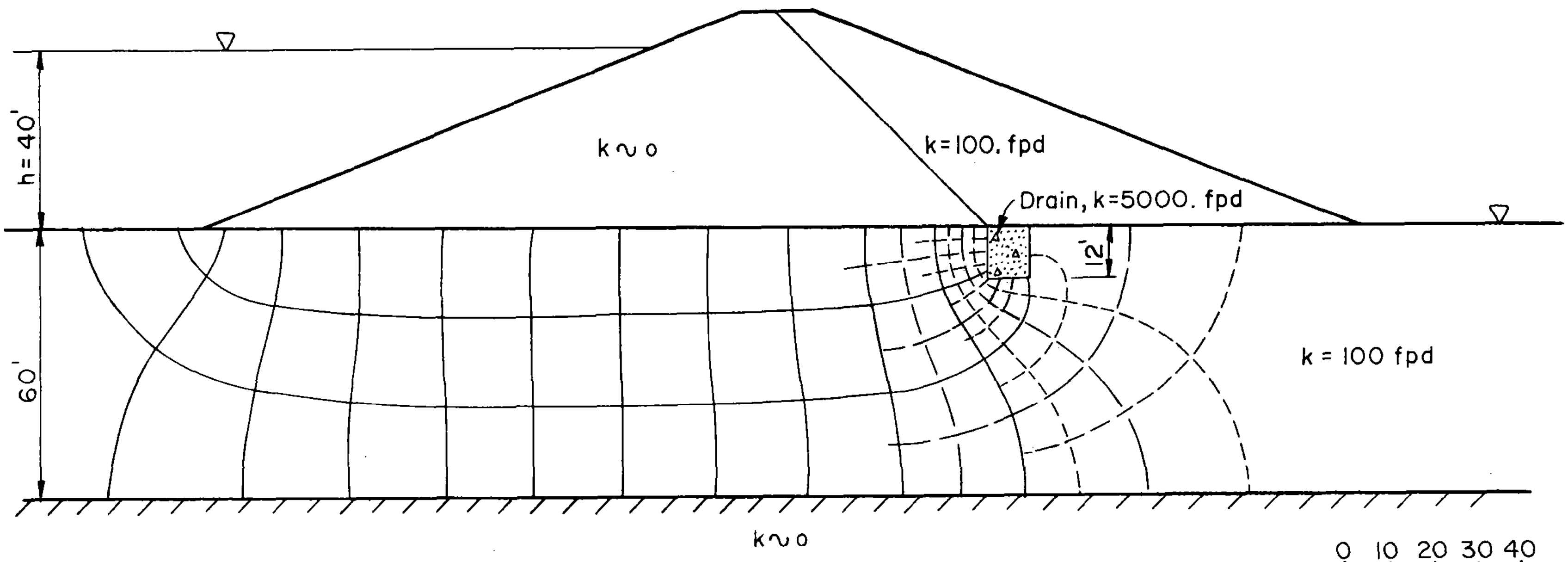


$$N_f = 4, \quad N_d = 7.25, \quad \Delta h = \frac{h}{N_d} = \frac{50}{7.25} = 6.9'$$

$$q = kh\frac{N_f}{N_d} = 0.01 \times 50 \times \frac{4}{7.25} = 0.28 \text{ cfd./ft. length of dam}$$

$$i_d = \frac{\Delta h}{L} = \frac{6.9}{2.5} = 2.8 \text{ (base of cutoff)}$$

Example 1. Seepage loss and discharge gradient.



Note: Since the permeability of the foundation is much more than that of the embankment, construct the flow net for the foundation only.

Estimate the discharge to the foundation drain.

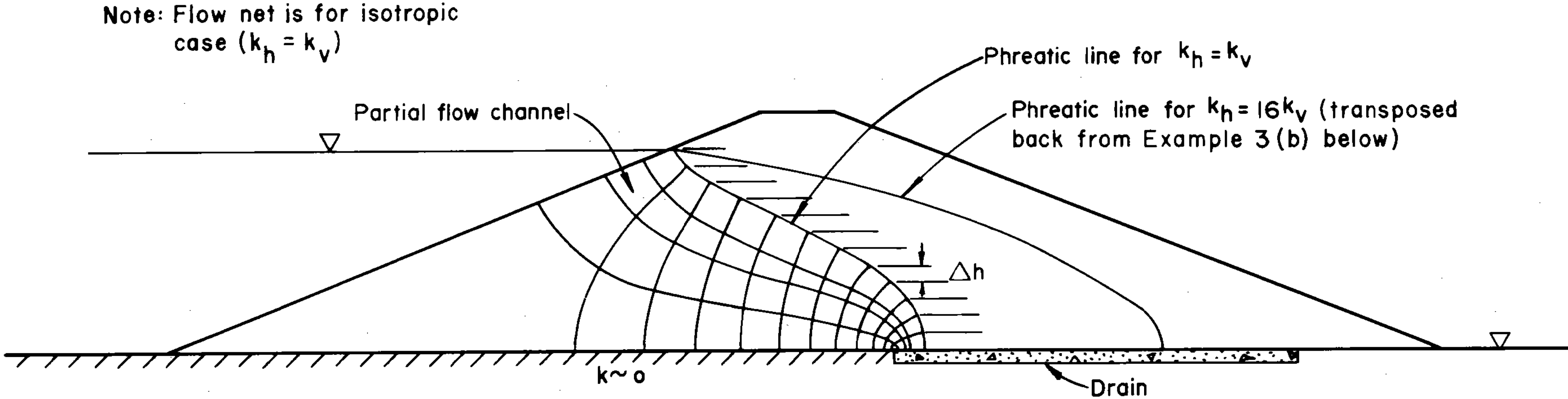
After subdivision, $N_f = 12$ and $N_d = 44$.

$$q = kh \frac{N_f}{N_d} \quad 100 \times 40 \times \frac{12}{44} = 1090 \text{ cfd/ft. of dam (total discharge per ft.)}$$

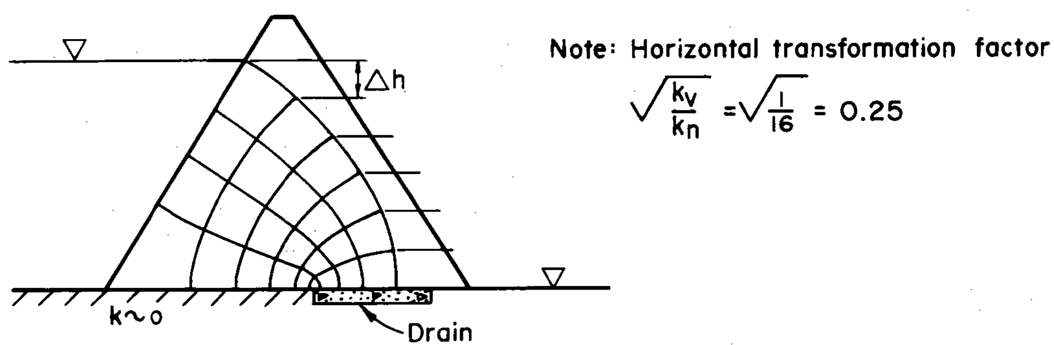
About 9.5 flow channels contact the drain.

$$\text{Discharge to the drain} = \frac{9.5}{12} \times 1090 = 860 \text{ cfd/ft. of dam length}$$

Example 2. Discharge to a drain

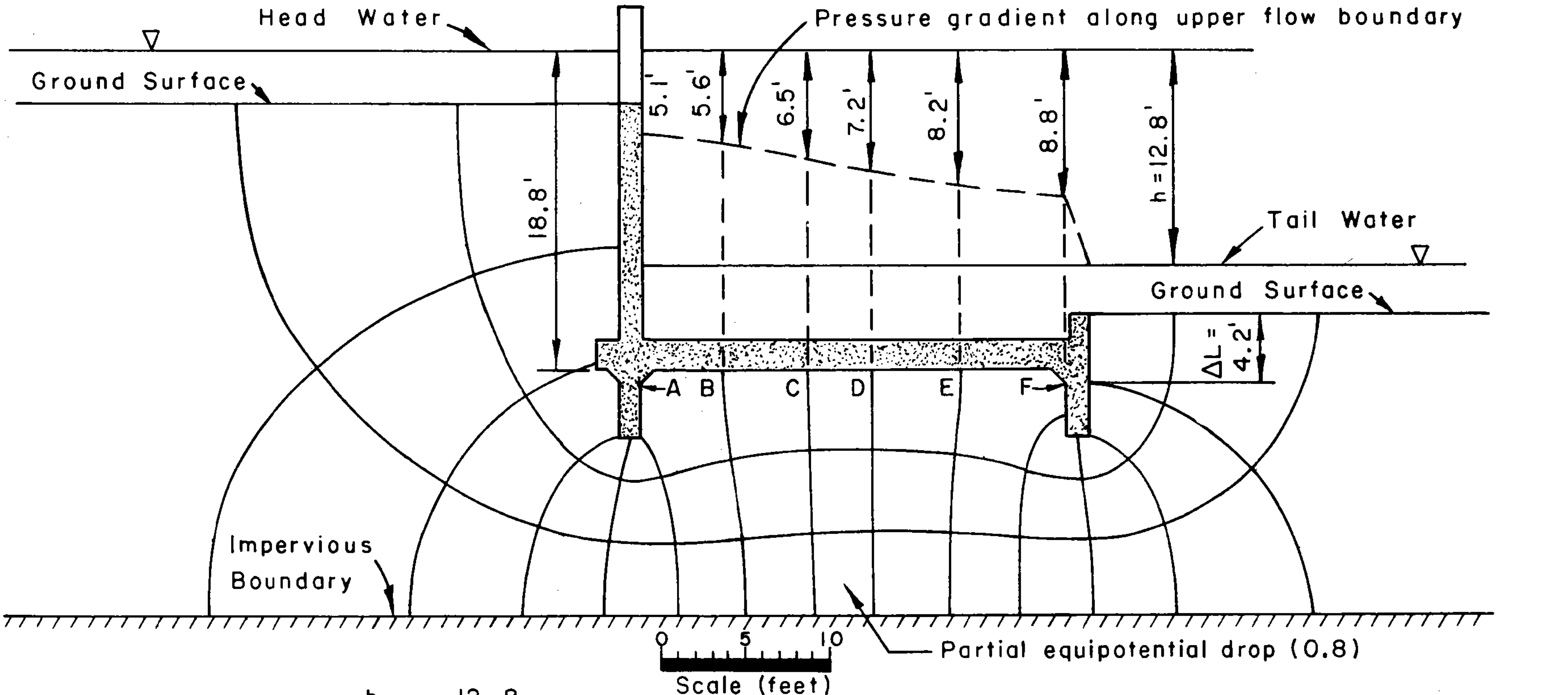


(a) True section



(b) Transformed section ($k_h = 16k_v$)

Example 3. Effect of anisotropy of embankment on the phreatic line.



$$N_d = 13.8; \quad \Delta h = \frac{h}{N_d} = \frac{12.8}{13.8} = 0.93'$$

Point designation	No. of drops to point	Head loss = (Δh x No. drops) (ft.)	Uplift pressure (18.8'- hd. loss) γ_w (p.s.f.)
A	5.5	5.1	855
B	6	5.6	824
C	7	6.5	768
D	7.8	7.2	725
E	8.8	8.2	662
F	9.5	8.8	624

Discharge gradient at toewall and transverse sill:

$$i_d = \frac{\Delta h}{\Delta L} = \frac{0.93}{4.2} = 0.22$$

Example 4. Concrete drop spillway – base uplift and discharge gradient.

X. Appendix - flow net examples

A. General

This section contains a number of flow nets in which equipotential lines were developed mainly by electric analog and flow lines by sketching. These nets provide information on how flow quantities, pressures, and gradients vary with changes in permeability ratios and various natural features and design features such as earth cutoffs, drains, and upstream blankets. These flow nets can only be used to solve actual problems when conditions and limitations at the actual structure are the same as in the examples.

The figures in this appendix, plus their conditions and limitations, are listed below for ease of reference.

Figure A-13. Embankment only. No core or drain. Steady seepage, 50% drawdown, and full drawdown conditions. $(k_h/k_v) = 1$.

Figure A-14. Plan view of seepage through periphery of reservoir and left abutment of dam.

Figure A-15. Embankment and two-layered foundation with high and low water tables. No cutoff or drain. Steady seepage condition. Different permeability values.

Figure A-16. Embankment and two-layered foundation with high and low water tables. Cutoff through upper layer. Steady seepage condition. Different permeability values.

B. Figures A-1 through A-12

The equipotential drops in these flow nets were selected arbitrarily, and the equipotential lines were developed by electric analog. Had these nets been developed entirely by sketching, a whole number of flow channels would have been chosen at the start. This procedure would have given partial equipotential drops.

These examples depict embankments resting on 40-foot thick permeable foundations that are underlain with impervious material. The embankment, upstream blanket (where applicable), and cutoff trench backfill are assumed to be impervious, and they are not considered in the flow nets. The base of the embankment (B) in each case is 220 feet long, and the length of the upstream blanket (where applicable) is 130 feet.

In most cases foundation permeability ratios (k_h/k_v) are 1, 25 and 100 where subscripts h and v denote horizontal and vertical directions respectively. The number of flow channels (N_f), number of equipotential drops (N_d), shape factors (N_f/N_d), and shortest flow path (ΔL) in which residual excess head (Δh) must be dissipated are shown for comparative purposes. The ΔL values are generally the flow distance from the last equipotential line shown to the downstream toe. However, in a few cases they are the flow distance to the bottom of the plunge basin or a point downstream from the toe.

Values of discharge gradient (i_d) in terms of net head (h) and rate of discharge (q) per foot of dam length in terms of kh or k'h are given in each figure. Equations 4a and 4b from Figure 1, Section IV, were used to develop i_d and q expressions. Discharge gradient $(i_d) = \Delta h/\Delta L$ but $\Delta h = h/N_d$; therefore, $i_d = h/(N_d \Delta L)$. The discharge gradients given on the figures are those which give the highest values from the last equipotential lines shown in the figures.

In applying these nets to actual situations, head (h) and embankment slopes may be varied so long as other limitations such as embankment base length, depth of pervious foundation, and permeability ratios are equivalent.

C. Figures A-13 through A-16

Flow nets in these figures are entirely different from the previous figures. Many of these nets were developed by electric analog for actual structure sites by spray painting a rigid insulating material with a graphite mixture to achieve variations in conductivity.

They provide an opportunity to observe seepage patterns and head relationships as water moves through embankment only and through embankment-foundation situations. Previous nets showed water percolating through foundations only. The effects of high and low water tables on a steady seepage condition are depicted. One figure simulates the movement of water through a pervious layer in the abutment of a dam and the periphery of the reservoir -- the layer being located at some distance below the reservoir water surface.

It is unlikely that many of these nets can be applied directly to the analysis of planned or existing structures. However, the examples shown do provide insight into hydraulic flow characteristics and pressure distribution pattern in earth masses.

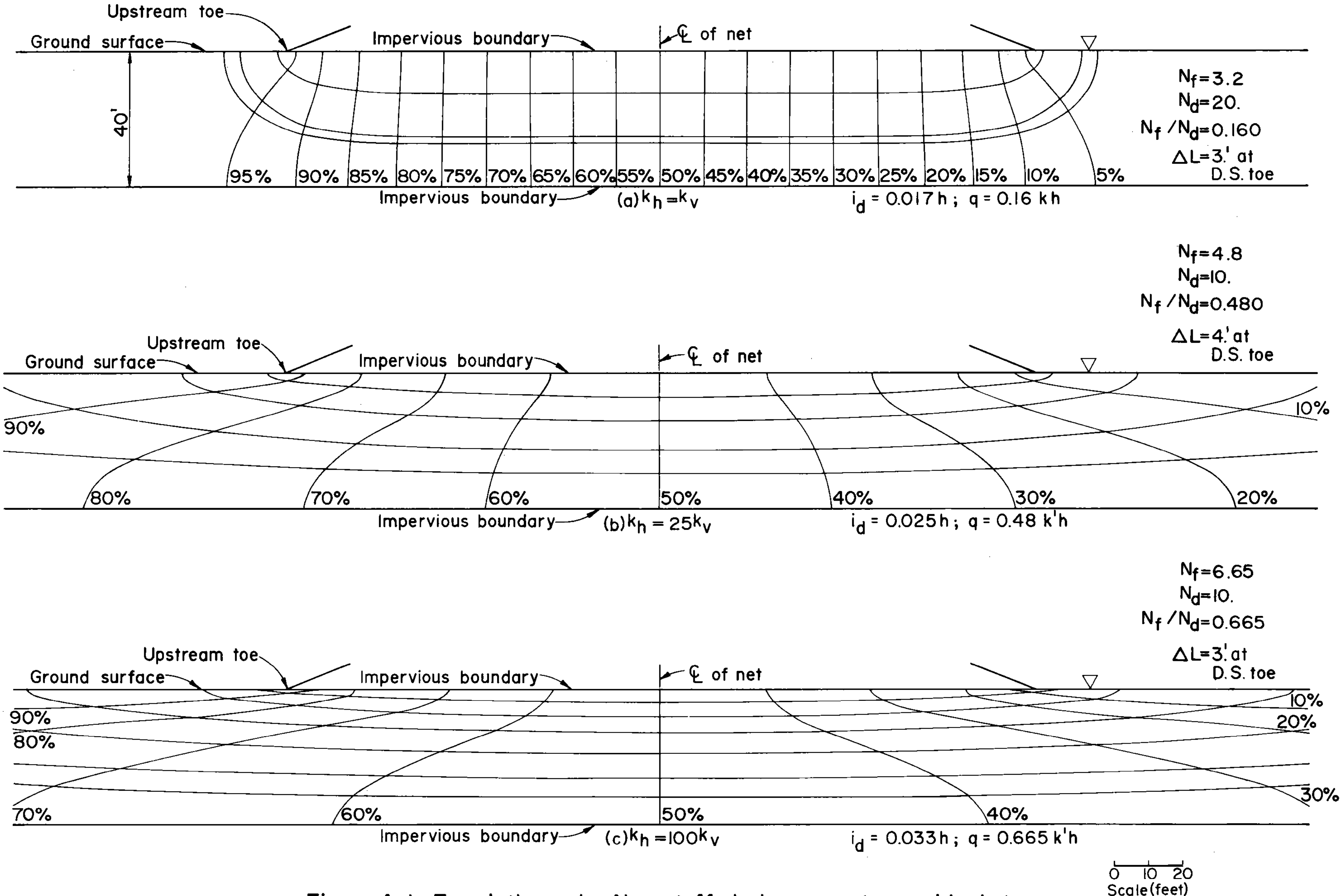
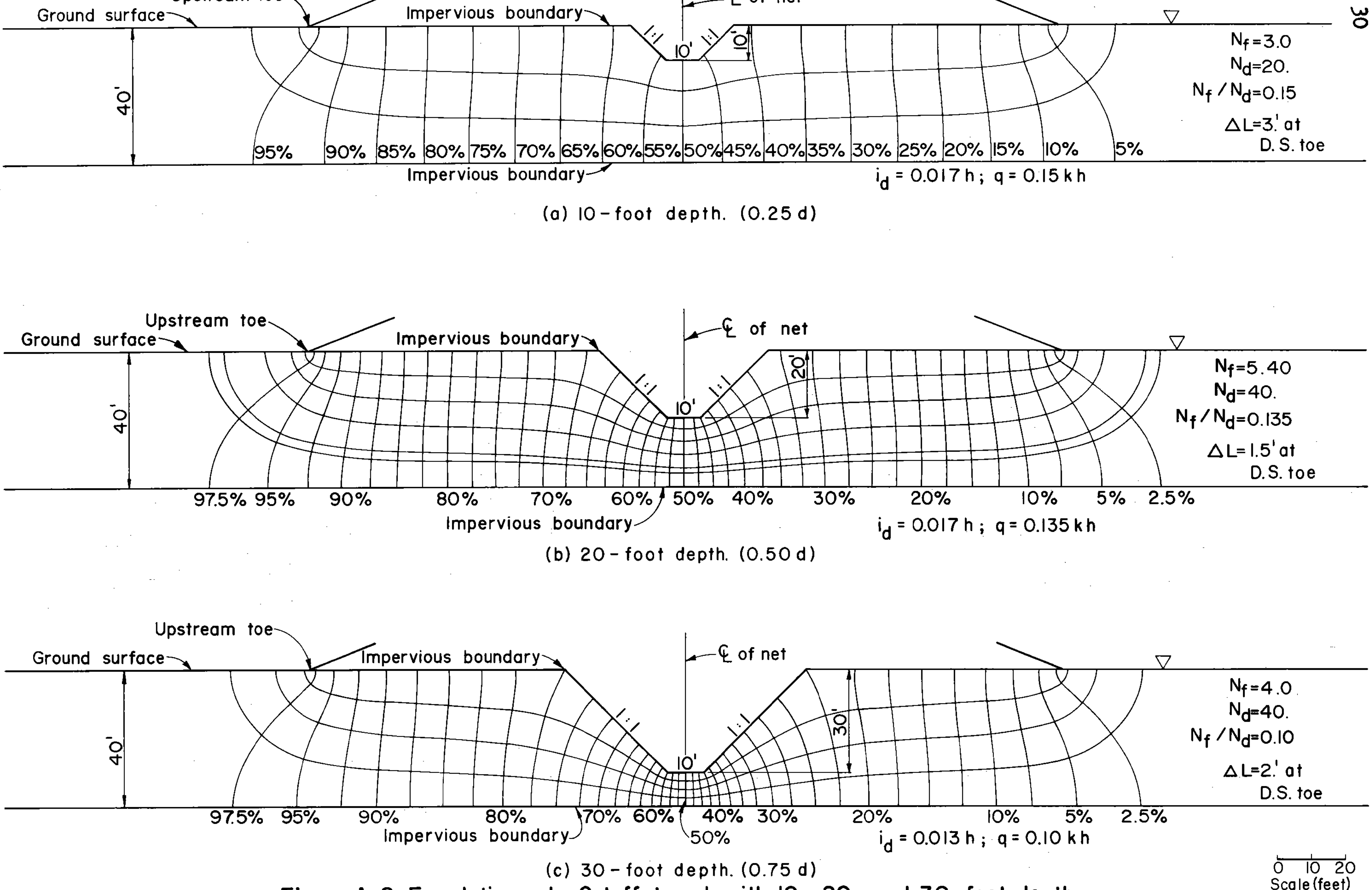


Figure A-1. Foundation only. No cutoff, drain, or upstream blanket.
(k_h / k_v)=1, 25, and 100



(a) 10 - foot depth. (0.25 d)

(b) 20 - foot depth. (0.50 d)

(c) 30 - foot depth. (0.75 d)

Figure A-2. Foundation only. Cutoff trench with 10-, 20-, and 30-foot depths. $(k_h / k_v) = 1$

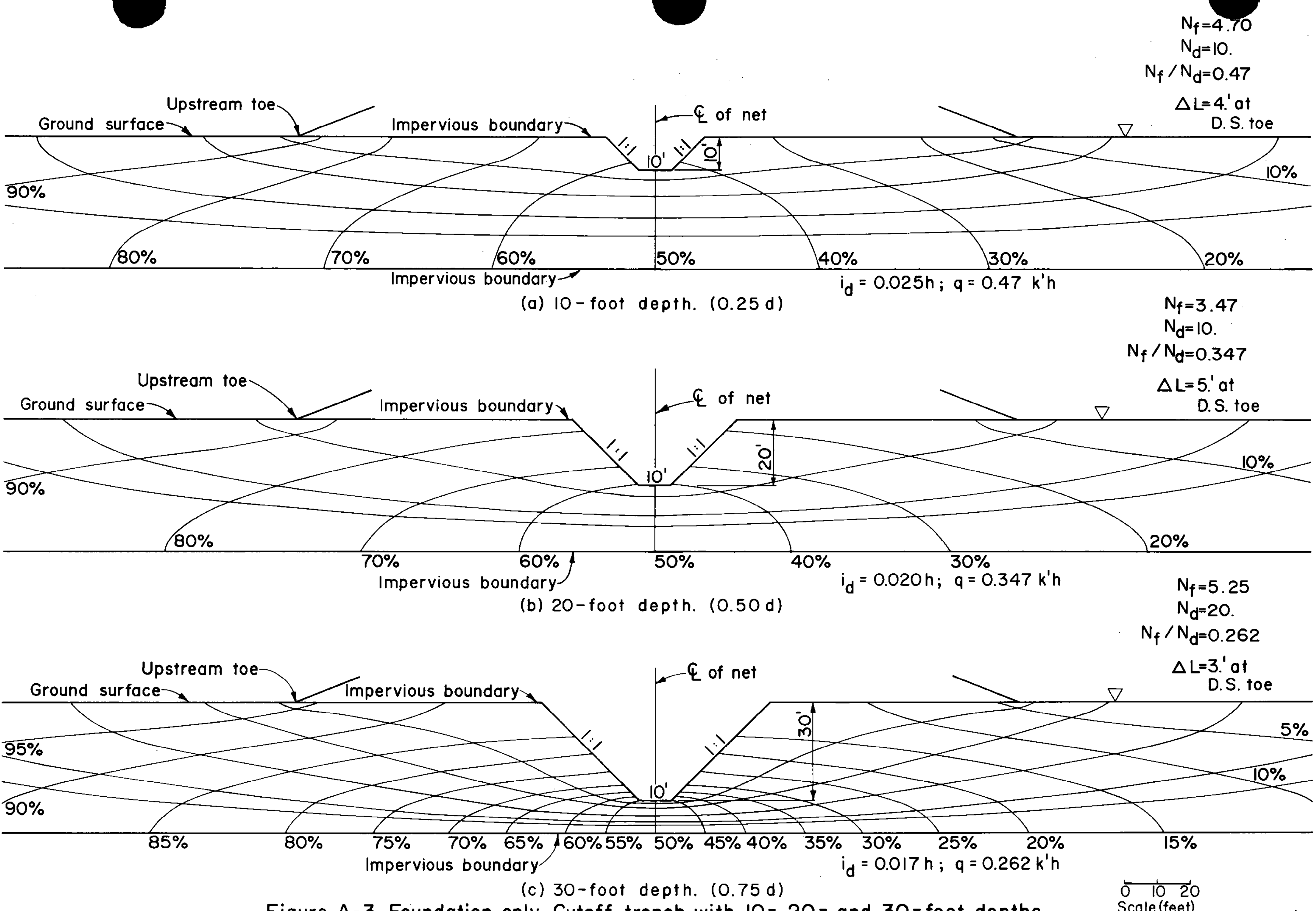


Figure A-3. Foundation only. Cutoff trench with 10-, 20-, and 30-foot depths.
$(k_h / k_v) = 25$

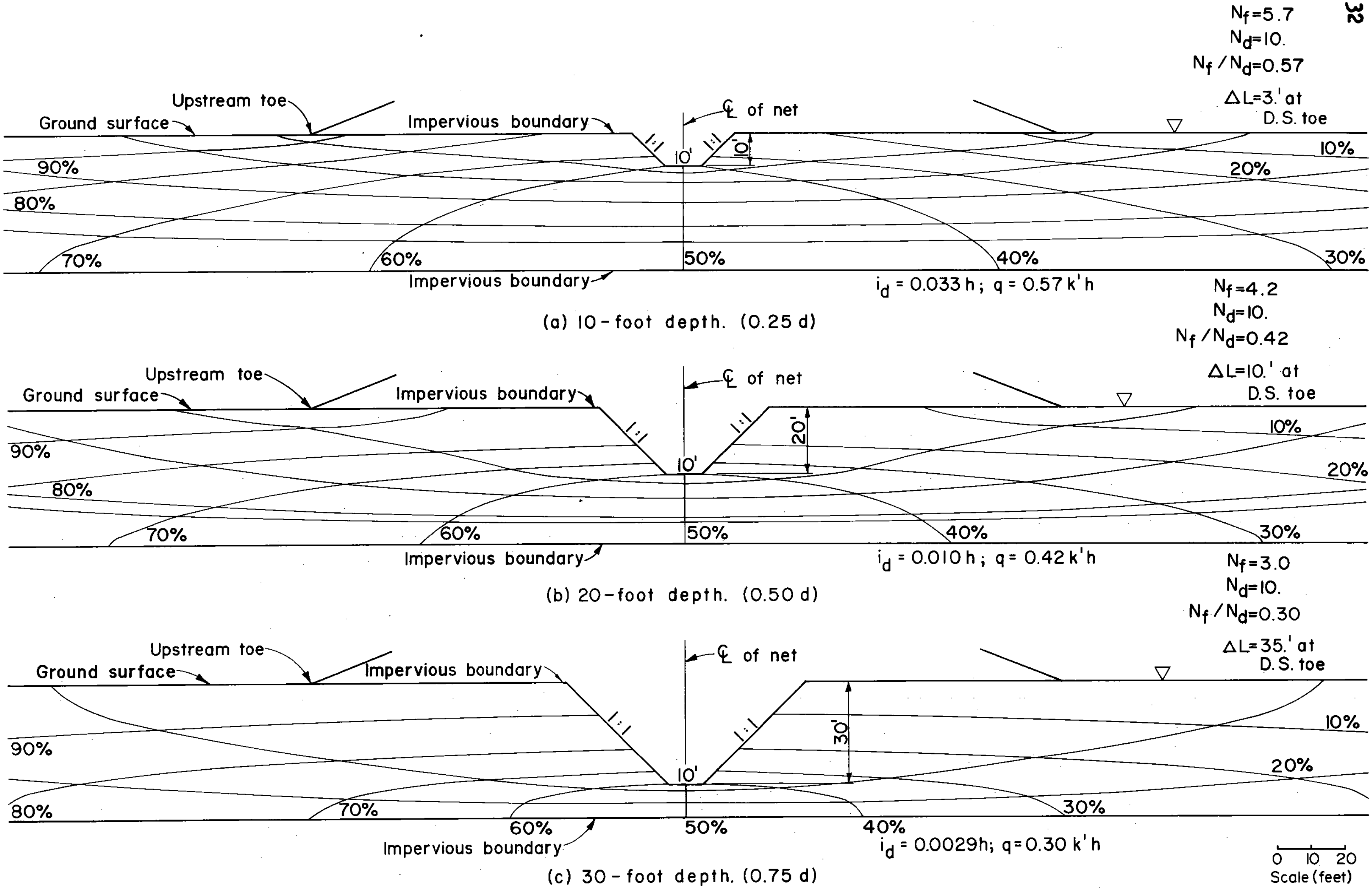


Figure A-4. Foundation only. Cutoff trench with 10-, 20-, and 30-foot depths.
$(k_h / k_v) = 100$

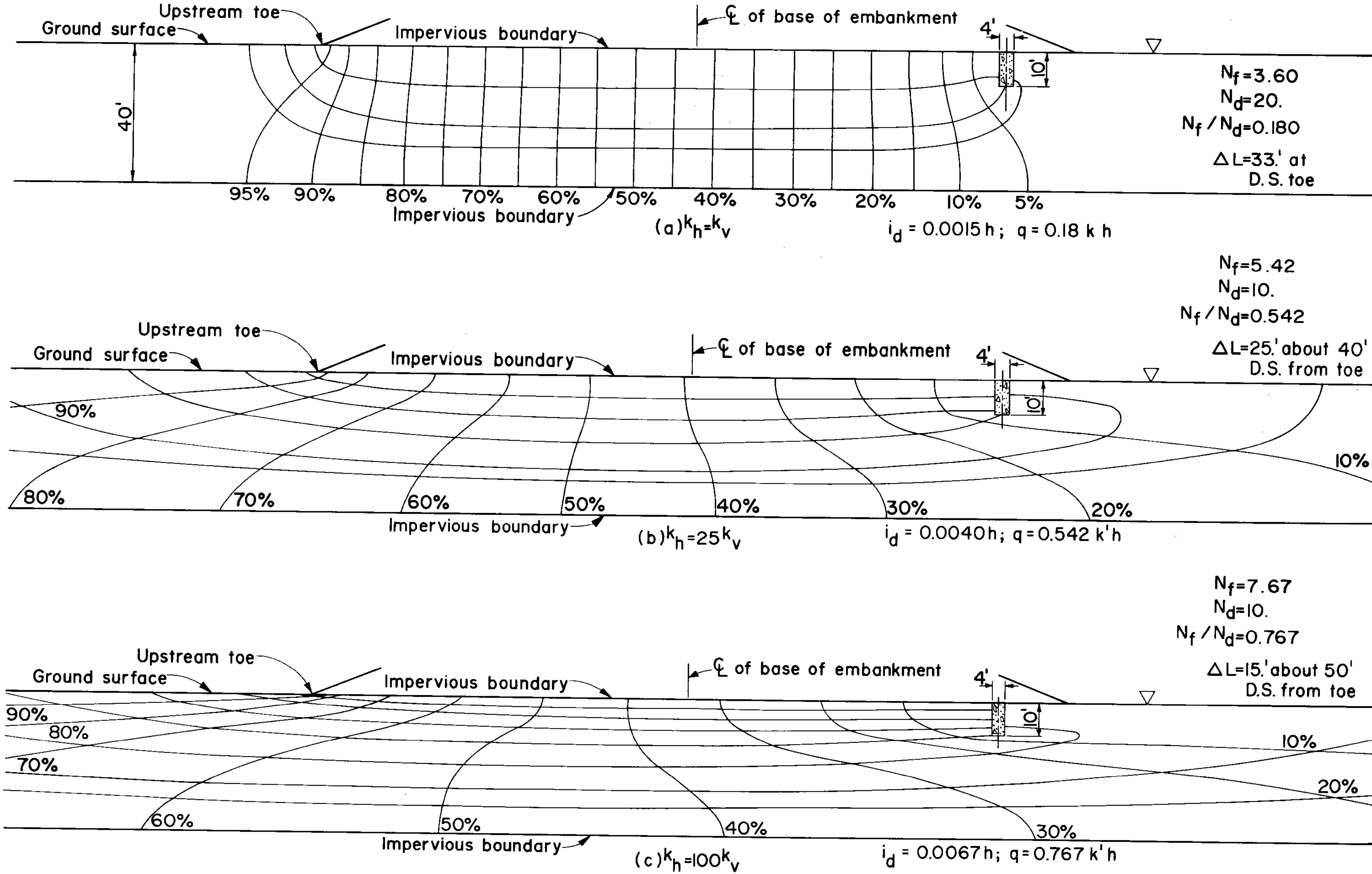


Figure A-5. Foundation only. Trench drain with 10-foot depth at c = 0.8 b (k_h / k_v)=1, 25, and 100

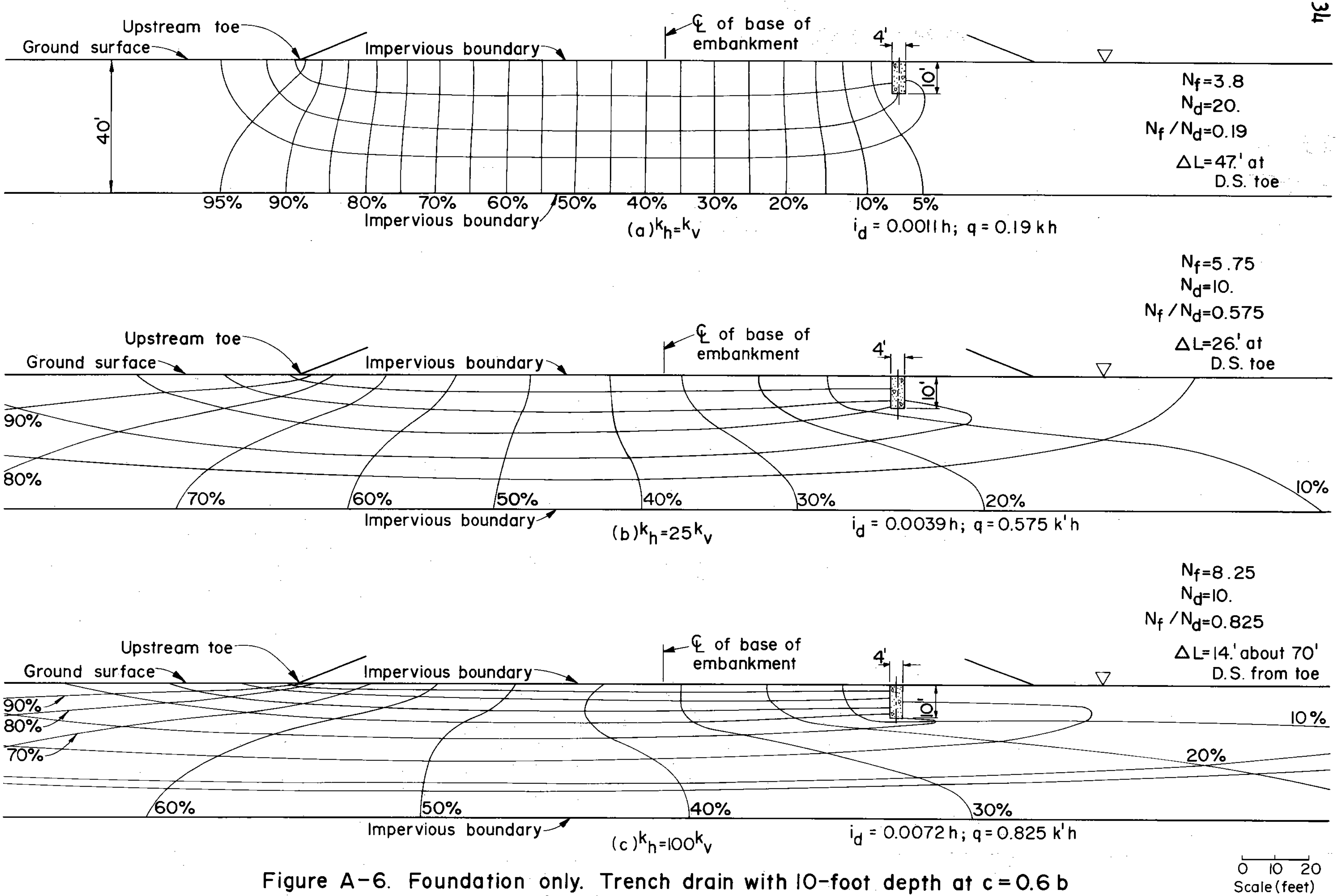


Figure A-6. Foundation only. Trench drain with 10-foot depth at c = 0.6 b
(k_h / k_v) = 1, 25, and 100

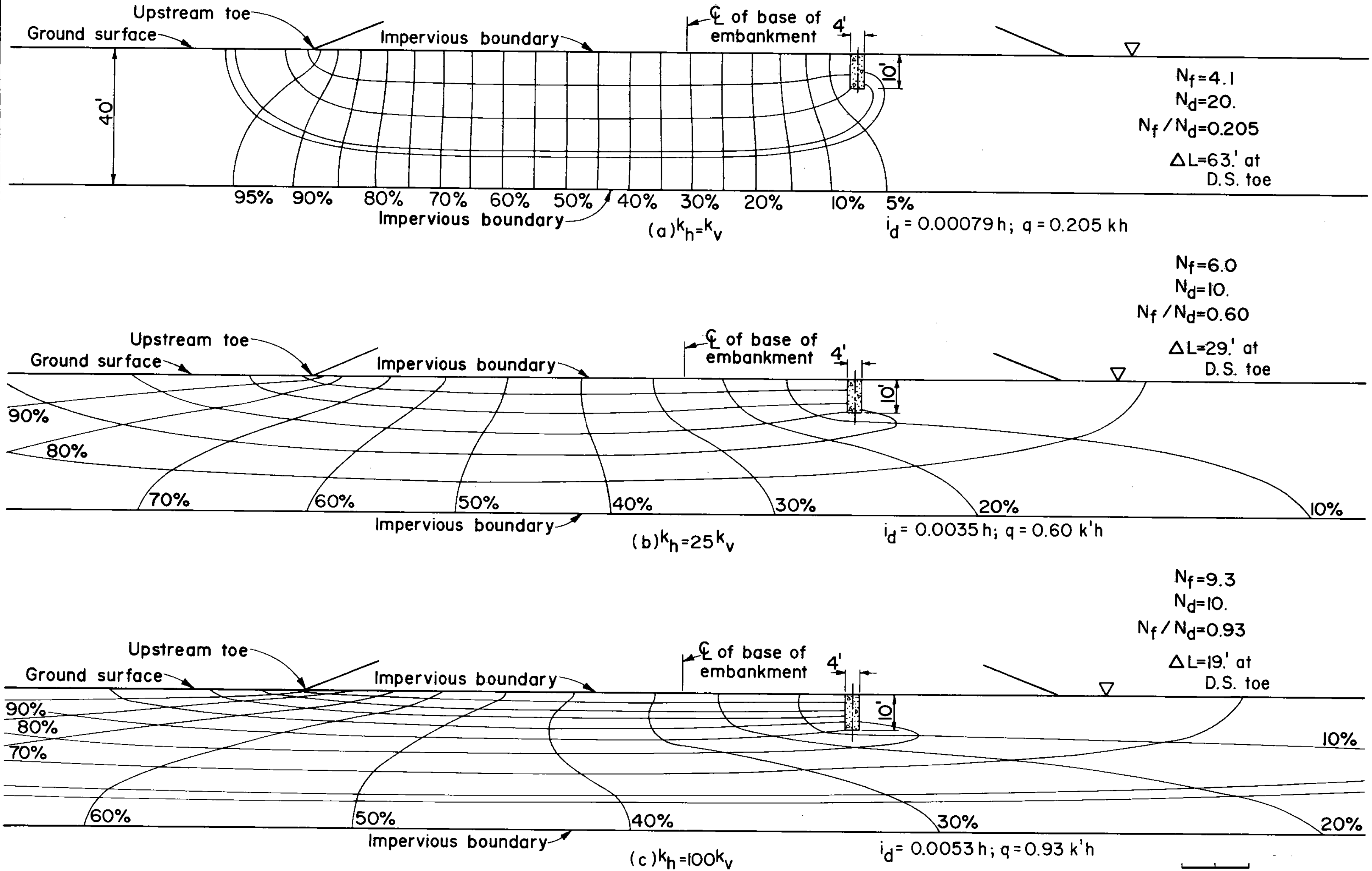


Figure A-7. Foundation only. Trench drain with 10-foot depth at c = 0.4 b
$(k_h / k_v) = 1, 25$, and 100

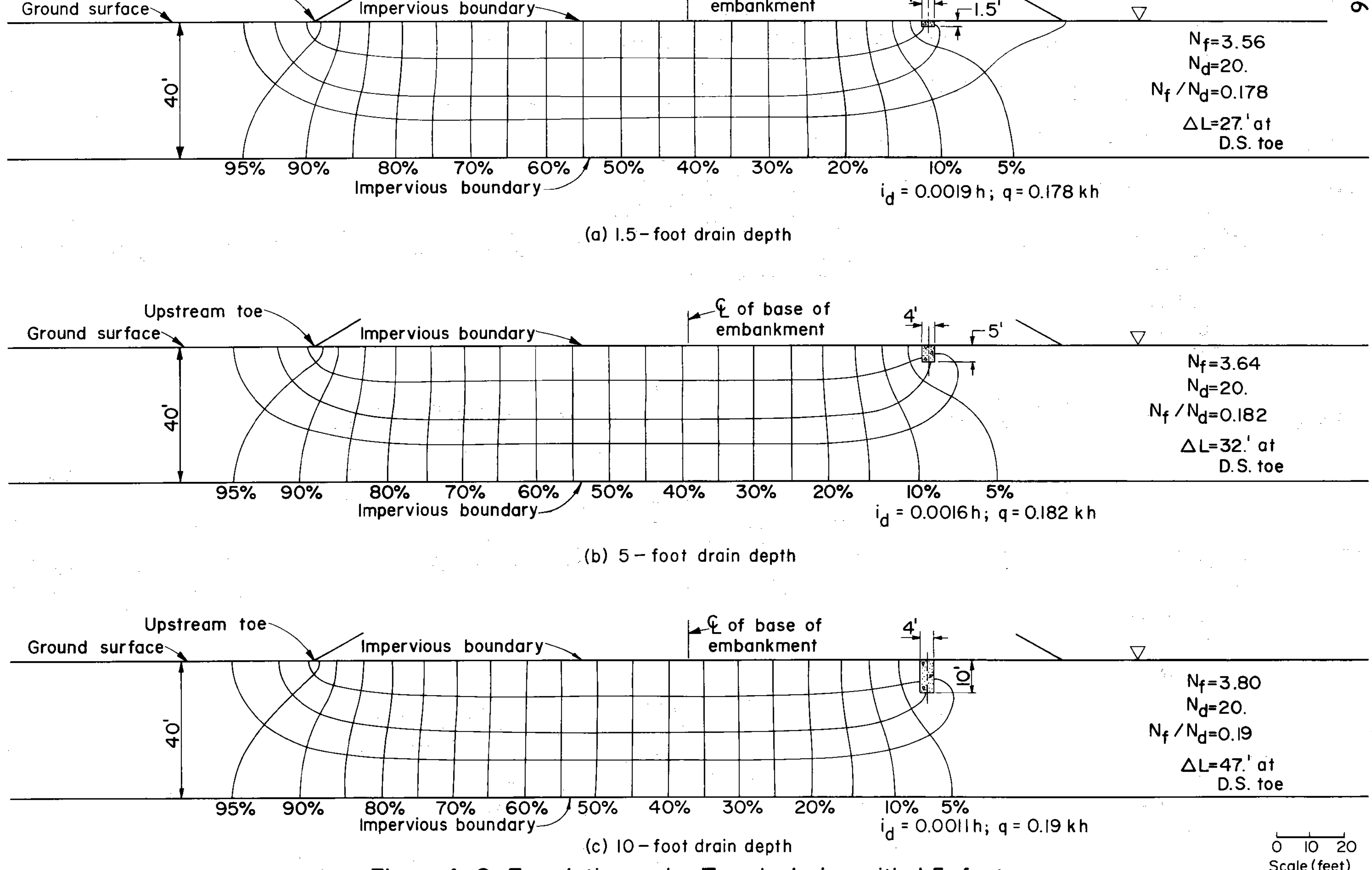


Figure A-8. Foundation only. Trench drains with 1.5-foot, 5-foot and 10-foot depths at c=0.6 b (k_h/k_v) = 1.

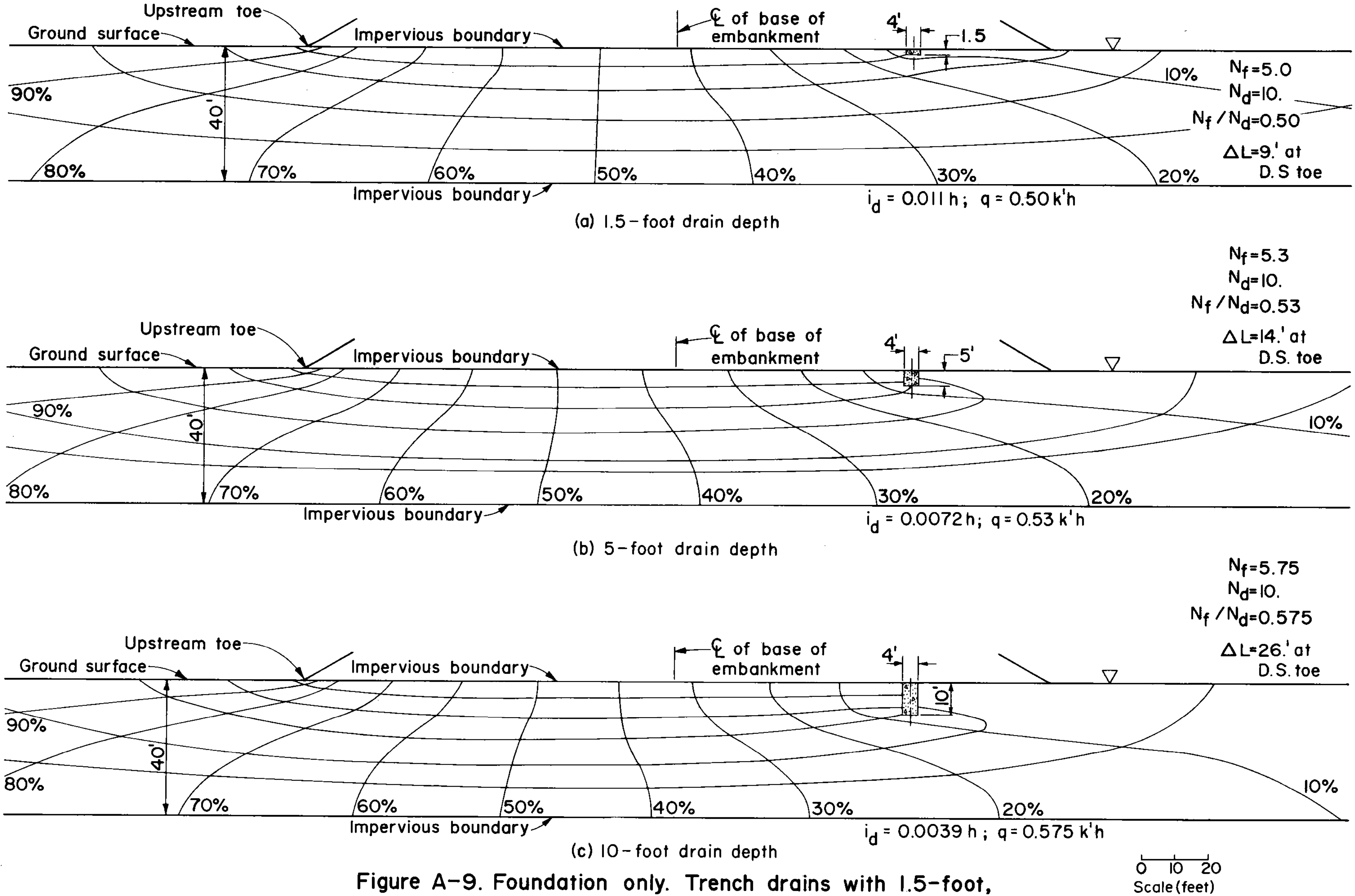


Figure A-9. Foundation only. Trench drains with 1.5-foot, 5-foot and 10-foot depths at c=0.6 b (k_h/k_v)=25.

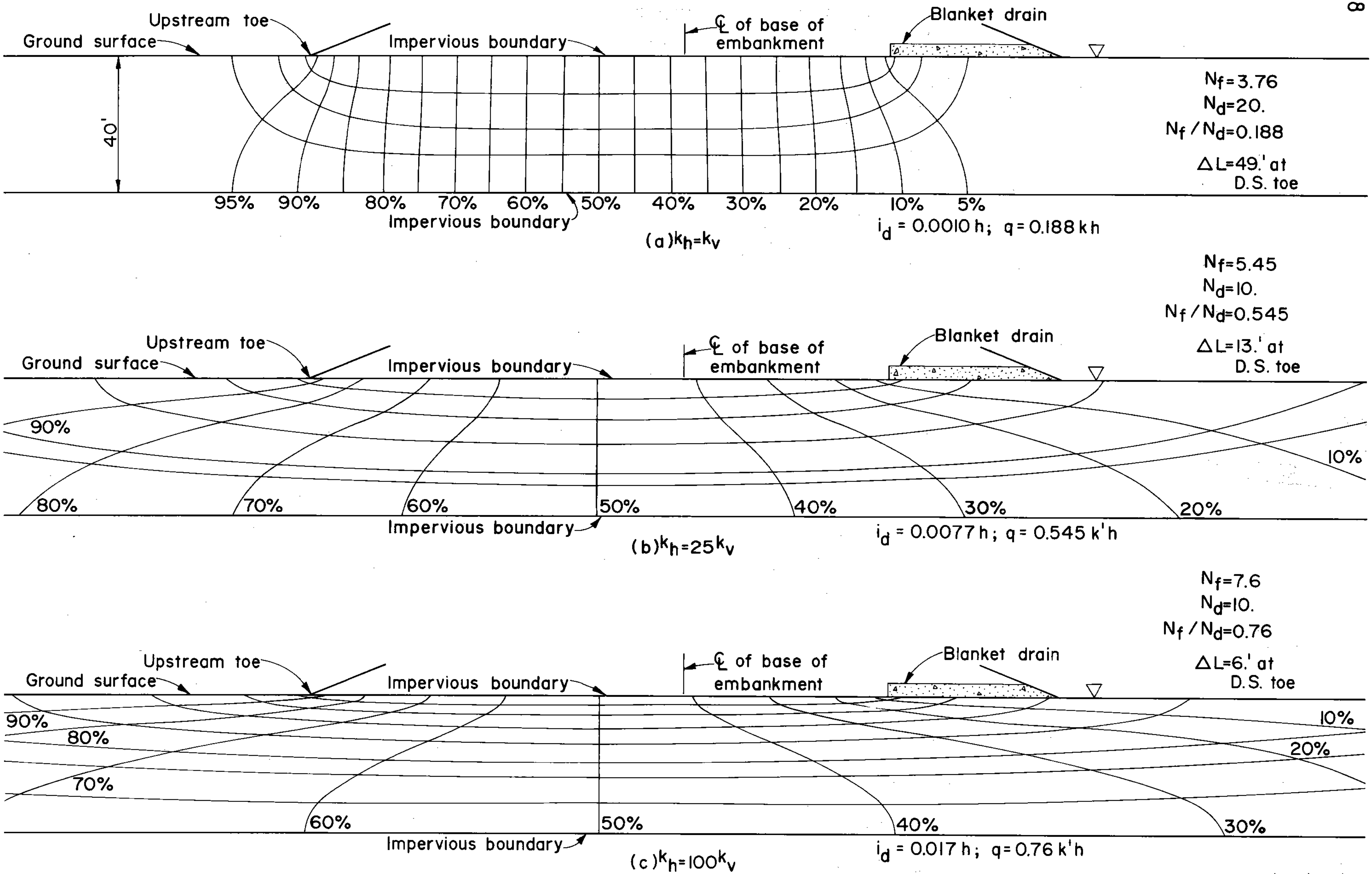


Figure A-10. Foundation only. Blanket drain from c=0.5b to downstream toe.
(k_h / k_v)=1, 25 and 100

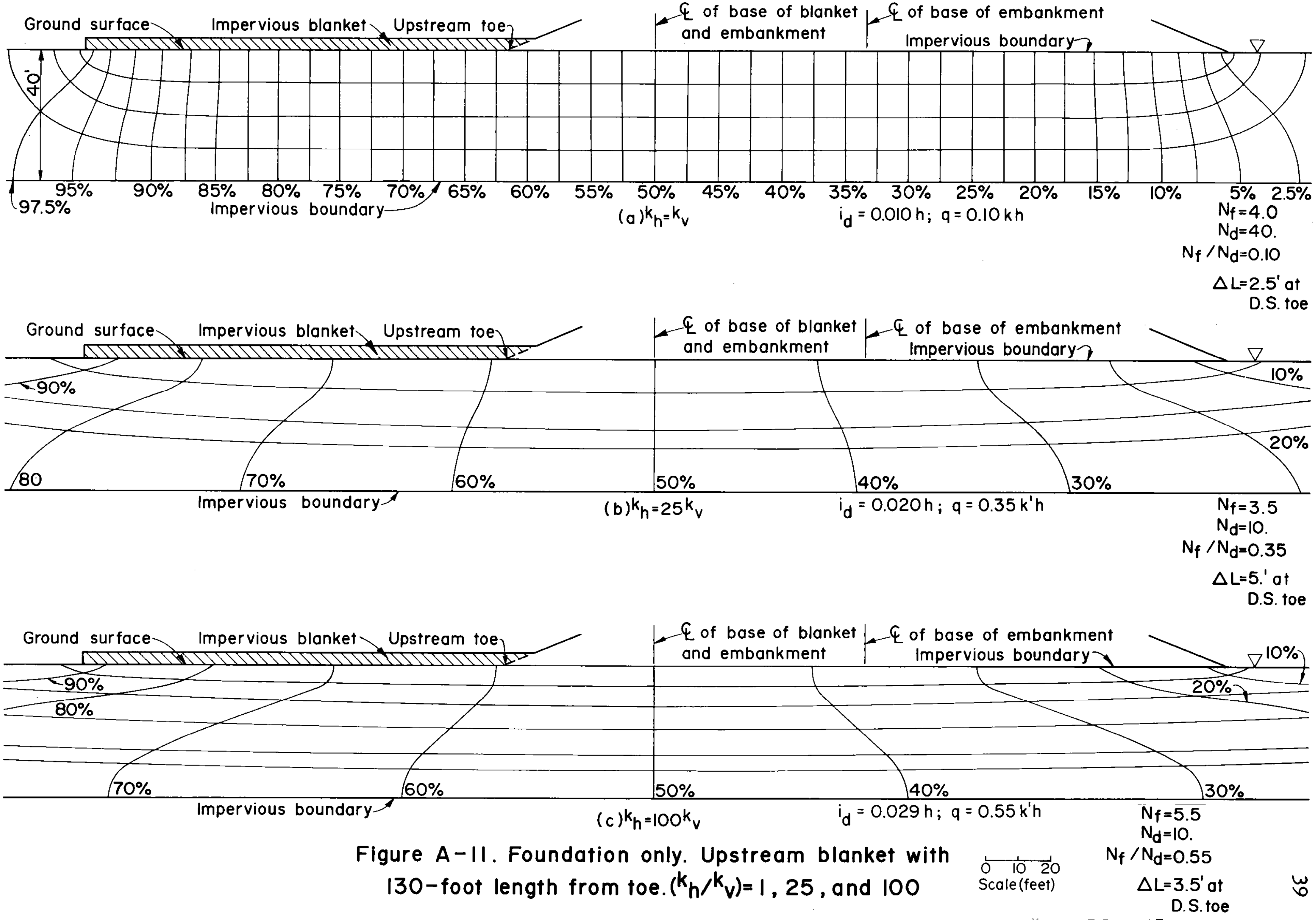


Figure A-11. Foundation only. Upstream blanket with 130-foot length from toe. $(k_h / k_v) = 1$, 25, and 100

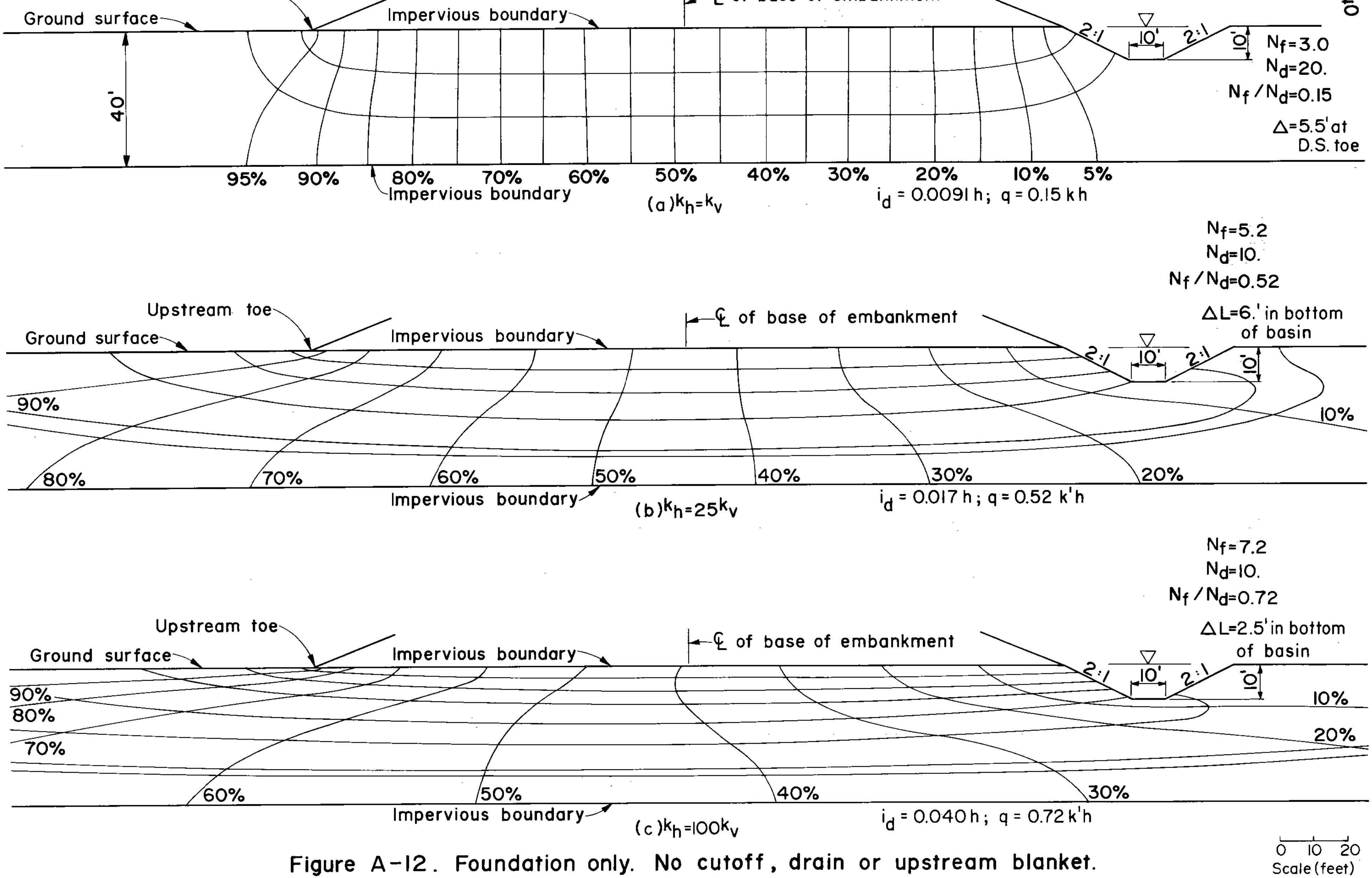


Figure A-12. Foundation only. No cutoff, drain or upstream blanket. Water-filled plunge basin with 10-foot depth. $(k_h/k_v)=1, 25,$ and 100.

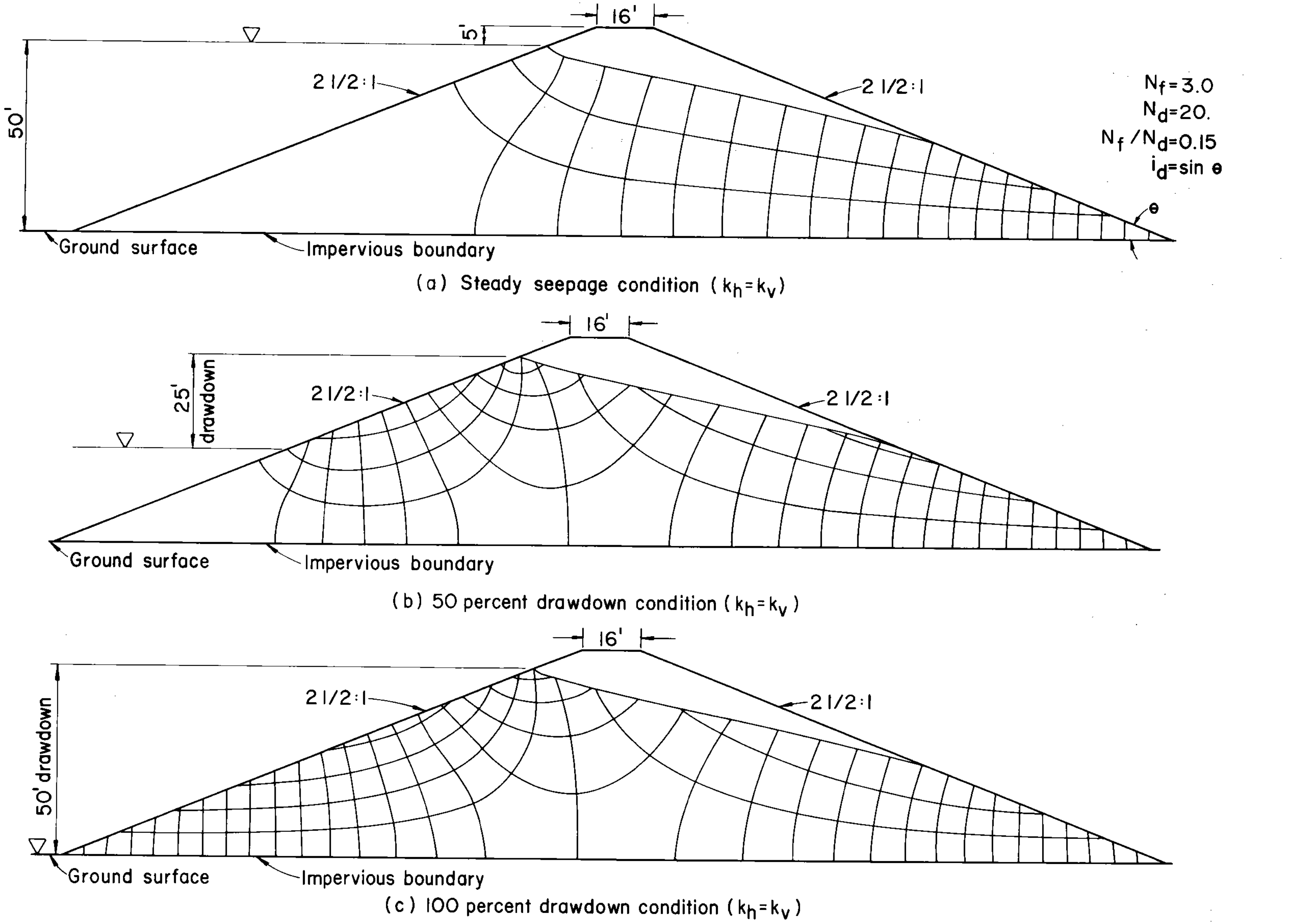


Figure A-13. Embankment only. No core or drain. Steady seepage, 50% drawdown, and full drawdown conditions (k_h/k_v) = 1.

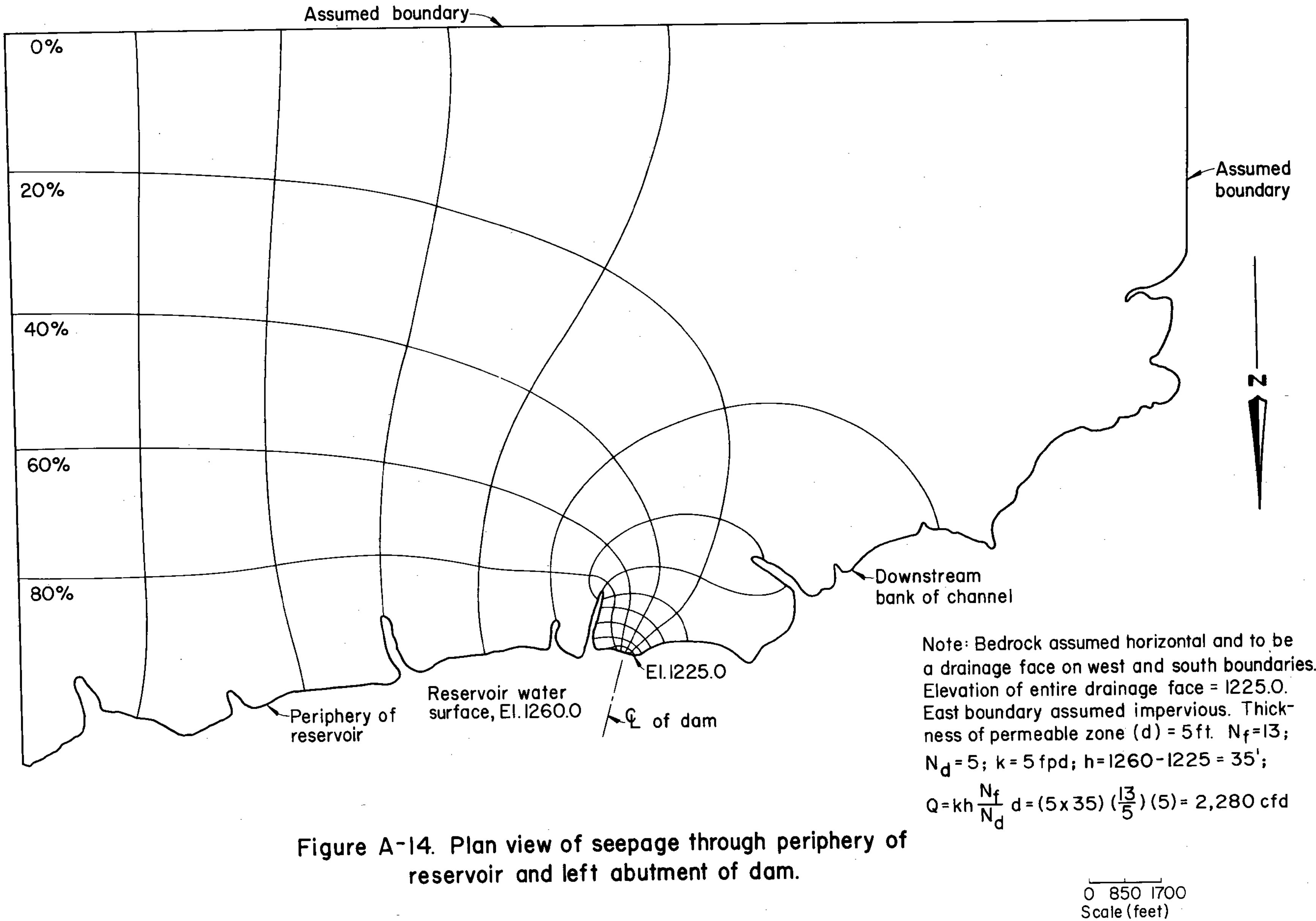
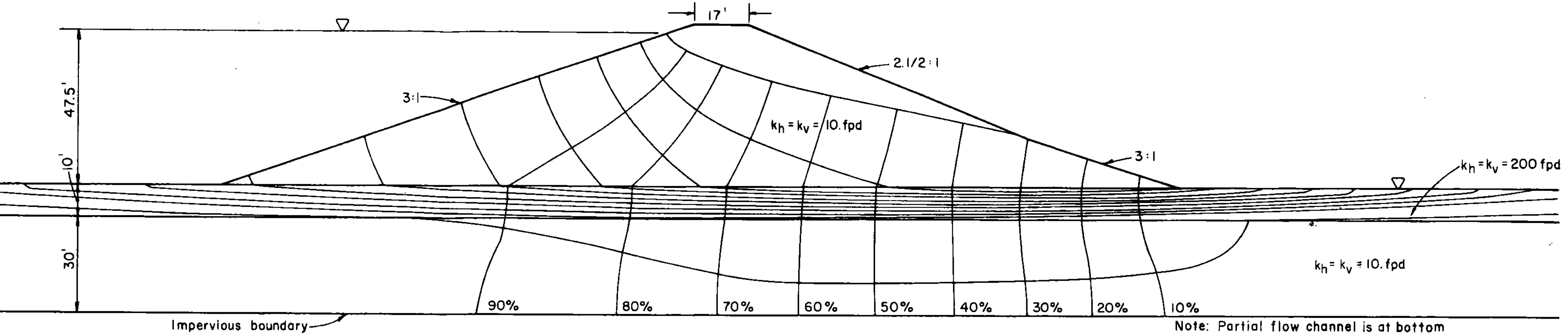
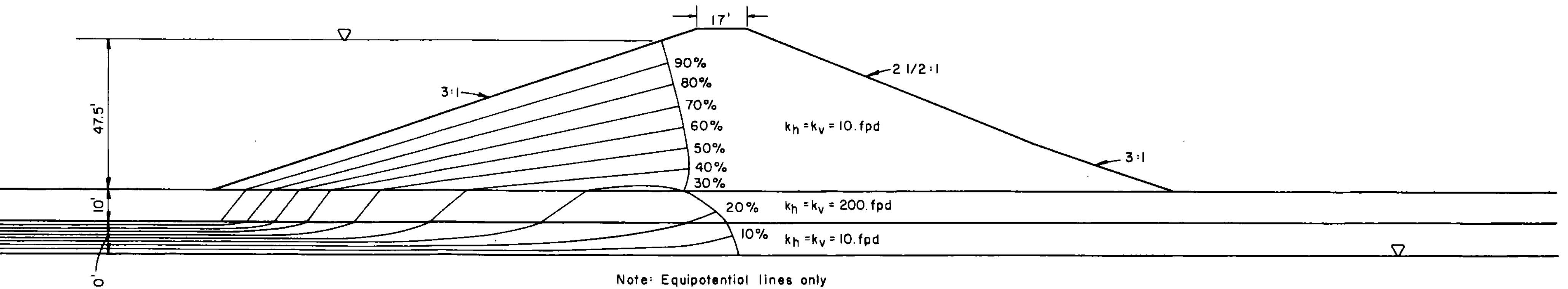


Figure A-14. Plan view of seepage through periphery of reservoir and left abutment of dam.



(a.) Steady seepage condition (high ground water level)



(b.) Steady seepage condition (low ground water level)

0 15 30
Scale (feet)

Figure A-15. Embankment and two-layered foundation with hig and low water tables. No cutoff or drain, steady seepag conditions, different permeability values.

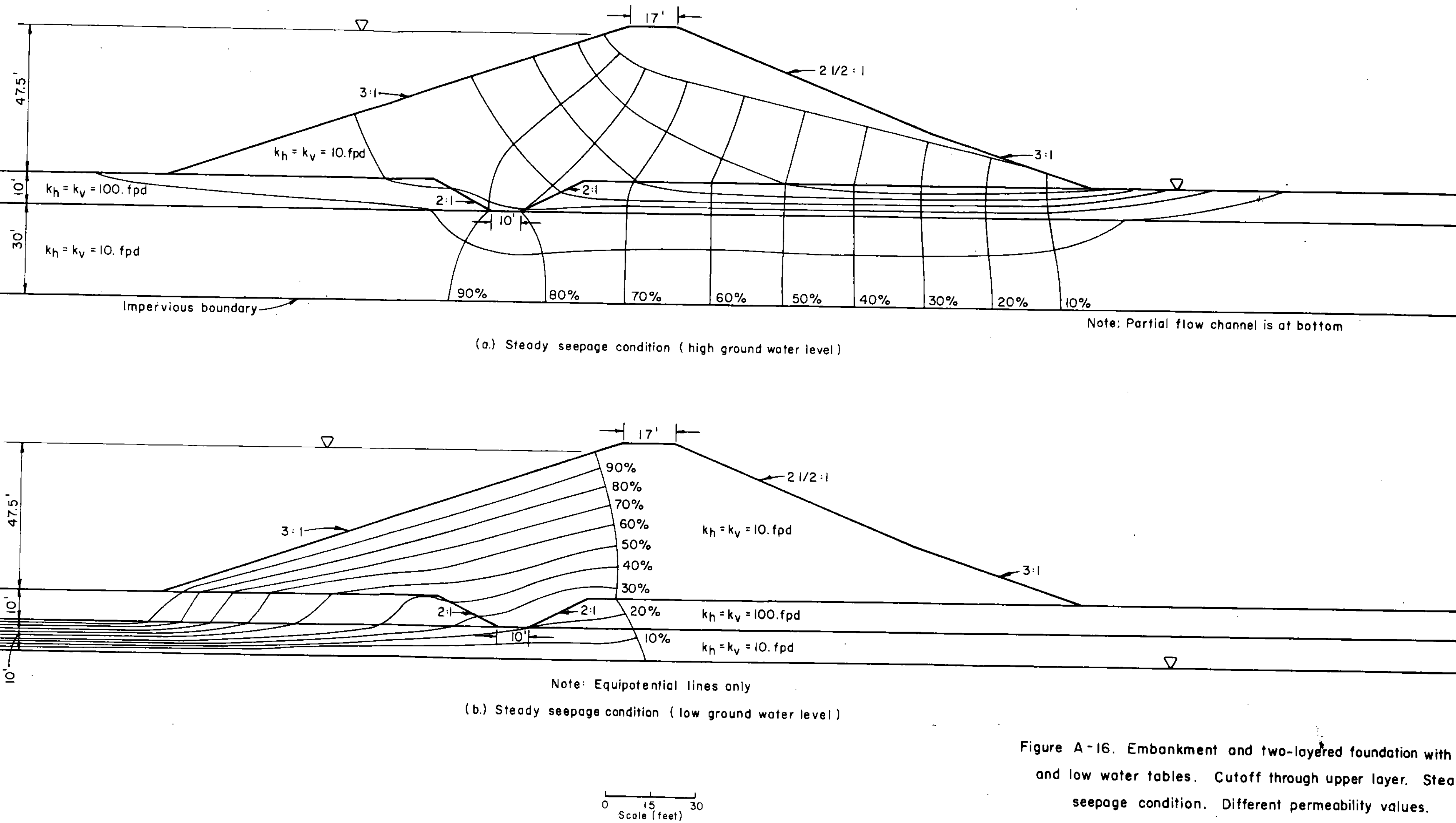


Figure A-16. Embankment and two-layered foundation with high and low water tables. Cutoff through upper layer. Steady seepage condition. Different permeability values.